sktest — Skewness and kurtosis tests for normality

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### Description

Title

For each variable in *varlist*, sktest presents a test for normality based on skewness and another based on kurtosis and then combines the two tests into an overall test statistic. sktest requires a minimum of eight observations to make its calculations. See [MV] mvtest normality for multivariate tests of normality.

# **Quick start**

```
Test for normality of v1 based on skewness and kurtosis
sktest v1
```

Separate tests for v1 and v2 sktest v1 v2

```
With frequency weights in wvar
sktest v1 v2 [fweight=wvar]
```

```
Suppress adjustment to the overall \chi^2 test sktest v1 v2, noadjust
```

### Menu

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### Syntax

sktest <i>varlist</i>	$\left[ if \right] \left[ in \right] \left[ weight \right] \left[ , options \right]$
options	Description
Main	
noadjust	do not adjust the overall $\chi^2$ test statistic and its <i>p</i> -value
nolstretch	do not automatically widen table for long variable names
collect is allowed; s	ee [U] 11.1.10 Prefix commands.

aweights and fweights are allowed; see [U] 11.1.6 weight. nolstretch does not appear in the dialog box.

# Option

Main

noadjust suppresses the empirical adjustment made by Royston (1991c) to the overall  $\chi^2$  test statistic and its *p*-value and presents the unaltered test as described by D'Agostino, Belanger, and D'Agostino (1990).

The following option is available with sktest but is not shown in the dialog box:

nolstretch; see [R] Estimation options.

## **Remarks and examples**

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Also see [R] **swilk** for the Shapiro–Wilk and Shapiro–Francia tests for normality. Those tests are, in general, preferred for nonaggregated data (Gould and Rogers 1991; Gould 1992; Royston 1991c). Moreover, a normal quantile plot should be used with any test for normality; see [R] **Diagnostic plots** for more information.

```
Example 1
```

Using our automobile dataset, we will test whether the variables mpg and trunk are normally distributed:

. use https:// (1978 automobi	-	ress.com/data/r	18/auto		
. sktest mpg t	trunk				
Skewness and h	kurtosis test	ts for normalit	у		
				Joint	test
Variable	Obs	Pr(skewness)	Pr(kurtosis)	Adj chi2(2)	Prob>chi2
mpg	74	0.0015	0.0804	10.95	0.0042
trunk	74	0.9115	0.0445	4.19	0.1228

We can reject the hypothesis that mpg is normally distributed, but we cannot reject the hypothesis that trunk is normally distributed, at least at the 12% level. The kurtosis for trunk is 2.19, as can be verified by issuing the command

```
. summarize trunk, detail
 (output omitted)
```

and the *p*-value of 0.0445 shown in the table above indicates that it is significantly different from the kurtosis of a normal distribution at the 5% significance level. However, on the basis of skewness alone, we cannot reject the hypothesis that trunk is normally distributed.

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#### Technical note

sktest implements the test as described by D'Agostino, Belanger, and D'Agostino (1990) but with the adjustment made by Royston (1991c). In the above example, if we had specified the noadjust option, the  $\chi^2$  values would have been 13.13 for mpg and 4.05 for trunk. With the adjustment, the  $\chi^2$  value might show as '.'. This result should be interpreted as an absurdly large number; the data are most certainly not normal.

# Stored results

sktest stores the following in r():

```
Scalarsr(N)number of observationsr(chi2)overall \chi^2r(p_skew)p-value for test based on skewnessr(p_kurt)p-value for test based on kurtosisr(p_chi2)p-value for overall \chi^2 testMatricesr(table)matrix of displayed results, one row per variable
```

# Methods and formulas

sktest implements the test described by D'Agostino, Belanger, and D'Agostino (1990) with the empirical correction developed by Royston (1991c).

Let  $g_1$  denote the coefficient of skewness and  $b_2$  denote the coefficient of kurtosis as calculated by summarize, and let n denote the sample size. If weights are specified, then  $g_1$ ,  $b_2$ , and ndenote the weighted coefficients of skewness and kurtosis and weighted sample size, respectively. See [R] summarize for the formulas for skewness and kurtosis.

To perform the test of skewness, we compute

$$Y = g_1 \left\{ \frac{(n+1)(n+3)}{6(n-2)} \right\}^{1/2}$$
  
$$\beta_2(g_1) = \frac{3(n^2 + 27n - 70)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)}$$
  
$$W^2 = -1 + \left[2 \left\{\beta_2(g_1) - 1\right\}\right]^{1/2}$$
  
$$\alpha = \left\{2/(W^2 - 1)\right\}^{1/2}$$

and

Then, the distribution of the test statistic

$$Z_{1} = \frac{1}{\sqrt{\ln W}} \ln \left[ Y/\alpha + \left\{ (Y/\alpha)^{2} + 1 \right\}^{1/2} \right]$$

is approximately standard normal under the null hypothesis that the data are distributed normally.

To perform the test of kurtosis, we compute

$$E(b_2) = \frac{3(n-1)}{n+1}$$

$$var(b_2) = \frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}$$

$$X = \{b_2 - E(b_2)\} / \sqrt{var(b_2)}$$

$$\sqrt{\beta_1(b_2)} = \frac{6(n^2 - 5n + 2)}{(n+7)(n+9)} \left\{\frac{6(n+3)(n+5)}{n(n-2)(n-3)}\right\}^{1/2}$$

$$A = 6 + \frac{8}{\sqrt{\beta_1(b_2)}} \left[\frac{2}{\sqrt{\beta_1(b_2)}} + \left\{1 + \frac{4}{\beta_1(b_2)}\right\}^{1/2}\right]$$

and

Then, the distribution of the test statistic

$$Z_2 = \frac{1}{\sqrt{2/(9A)}} \left[ \left( 1 - \frac{2}{9A} \right) - \left\{ \frac{1 - 2/A}{1 + X\sqrt{2/(A-4)}} \right\}^{1/3} \right]$$

is approximately standard normal under the null hypothesis that the data are distributed normally.

D'Agostino, Balanger, and D'Agostino Jr.'s omnibus test of normality uses the statistic

$$K^2 = Z_1^2 + Z_2^2$$

which has approximately a  $\chi^2$  distribution with 2 degrees of freedom under the null of normality.

Royston (1991c) proposed the following adjustment to the test of normality, which sktest uses by default. Let  $\Phi(x)$  denote the cumulative standard normal distribution function for x, and let  $\Phi^{-1}(p)$  denote the inverse cumulative standard normal function [that is,  $x = \Phi^{-1} {\Phi(x)}$ ]. Define the following terms:

$$Z_{c} = -\Phi^{-1} \left\{ \exp\left(-\frac{1}{2}K^{2}\right) \right\}$$

$$Z_{t} = 0.55n^{0.2} - 0.21$$

$$a_{1} = (-5 + 3.46 \ln n) \exp(-1.37 \ln n)$$

$$b_{1} = 1 + (0.854 - 0.148 \ln n) \exp(-0.55 \ln n)$$

$$a_{2} = a_{1} - \left\{ 2.13/(1 - 2.37 \ln n) \right\} Z_{t}$$

$$b_{2} = 2.13/(1 - 2.37 \ln n) + b1$$

and

If  $Z_c < -1$  set  $Z = Z_c$ ; else if  $Z_c < Z_t$  set  $Z = a_1 + b_1 Z_c$ ; else set  $Z = a_2 + b_2 Z_c$ . Define  $P = 1 - \Phi(Z)$ . Then,  $K^2 = -2 \ln P$  is approximately distributed  $\chi^2$  with 2 degrees of freedom.

The relative merits of the skewness and kurtosis test versus the Shapiro–Wilk and Shapiro–Francia tests have been a subject of debate. Our recommendation is to use the Shapiro–Francia test whenever possible, that is, whenever dealing with nonaggregated or ungrouped data (Gould and Rogers 1991; Gould 1992); see [R] swilk. If normality is rejected, use sktest to determine the source of the problem.

As both D'Agostino, Belanger, and D'Agostino (1990) and Royston (1991d) mention, researchers should also examine the normal quantile plot to determine normality rather than blindly relying on a few test statistics. See the qnorm command documented in [R] **Diagnostic plots** for more information on normal quantile plots.

sktest is similar in spirit to the Jarque-Bera (1987) test of normality. The Jarque-Bera test statistic is also calculated from the sample skewness and kurtosis, though it is based on asymptotic standard errors with no corrections for sample size. In effect, sktest offers two adjustments for sample size, that of Royston (1991c) and that of D'Agostino, Belanger, and D'Agostino (1990).

### Acknowledgments

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### Also see

- [R] Diagnostic plots Distributional diagnostic plots
- [R] ladder Ladder of powers
- [R] **lv** Letter-value displays
- [R] swilk Shapiro-Wilk and Shapiro-Francia tests for normality
- [MV] mvtest normality Multivariate normality tests

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