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**intreg** — Interval regression

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# **Description**

intreg fits a linear model with an outcome measured as point data, interval data, left-censored data, or right-censored data. As such, it is a generalization of the model fit by tobit.

# **Quick start**

Regression on x1 and x2 of an interval-measured dependent variable with lower endpoint y\_lower and upper endpoint y\_upper

```
intreg y_lower y_upper x1 x2
```

With robust standard errors

```
intreg y_lower y_upper x1 x2, vce(robust)
```

Model heteroskedasticity in the conditional variance as a function of x3

```
intreg y_lower y_upper x1 x2, het(x3)
```

Adjust for complex survey design using svyset data

```
svy: intreg y_lower y_upper x1 x2
```

# Menu

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options

```
intreg depvar_1 depvar_2 [indepvars] [if] [in] [weight] [, options]
```

Description

 $depvar_1$  and  $depvar_2$  should have the following form:

Type of data		$depvar_1$	$depvar_2$
point data	a = [a, a]	a	$\overline{a}$
interval data	[a,b]	a	b
left-censored data	$(-\infty, b]$	•	b
right-censored data	$[a, +\infty)$	a	
missing			

Model	
<u>nocons</u> tant	suppress constant term
<pre>het(varlist[, noconstant])</pre>	independent variables to model the variance; use noconstant to suppress constant term
<pre>offset(varname)</pre>	include varname in model with coefficient constrained to 1
<pre>constraints(constraints)</pre>	apply specified linear constraints
SE/Robust	
vce(vcetype)	<pre>vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife</pre>
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
<u>nocnsr</u> eport	do not display constraints
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
maximize_options	control the maximization process; seldom used
<u>col</u> linear	keep collinear variables
<u>coefl</u> egend	display legend instead of statistics
. 1 1 1	11 FIR 44 42 F 4 111

indepvars and varlist may contain factor variables; see [U] 11.4.3 Factor variables.

depvar<sub>1</sub>, depvar<sub>2</sub>, indepvars, and varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bayes, bootstrap, by, collect, fmm, fp, jackknife, mfp, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: intreg and [FMM] fmm: intreg.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

aweights are not allowed with the jackknife prefix; see [R] jackknife.

vce() and weights are not allowed with the svy prefix; see [SVY] svy.

aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

# **Options**

```
Model
 noconstant; see [R] Estimation options.
het(varlist [, noconstant]) specifies that the logarithm of the standard deviation be modeled as a linear combination of varlist. The constant is included unless noconstant is specified.
 offset(varname), constraints(constraints); see [R] Estimation options.
                   SE/Robust
 vce (vcetype) specifies the type of standard error reported, which includes types that are derived from
        asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that
        allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods
        (bootstrap, jackknife); see [R] vce_option.
                 Reporting
 level(#), nocnsreport; see [R] Estimation options.
 display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
         allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt),
         sformat(% fmt), and nolstretch; see [R] Estimation options.
                Maximization
 maximize_options: difficult, technique(algorithm_spec), iterate(#), |no|log, trace,
        gradient, showstep, <a href="hessian">hessian</a>, <a href="showtolerance">showtolerance</a>, <a href="tolerance">tolerance</a>(#), <a href="tolerance">ltolerance</a>(#), <a 
        <u>nrtolerance(#), nonrtolerance, and from(init_specs)</u>; see [R] Maximize. These options are
        seldom used.
```

Setting the optimization type to technique(bhhh) resets the default *vcetype* to vce(opg).

The following options are available with intreg but are not shown in the dialog box:

collinear, coeflegend; see [R] Estimation options.

# Remarks and examples

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intreg fits a linear model to an outcome that may be either observed exactly or unobserved but known to fall within some interval. The values of the outcome variable may be observed (point data), unobserved but known to fall within an interval with fixed endpoints (interval-censored data), unobserved but known to fall within an interval that has a fixed upper endpoint (left-censored data), or unobserved but known to fall within an interval that has a fixed lower endpoint (right-censored data). Such censored data arise naturally in many contexts, such as wage data. Often, you know only that, for example, a person's salary is between \$30,000 and \$40,000.

The interval regression model fit by intreg is a generalization of the models fit by tobit because it extends censoring beyond left-censored data or right-censored data; see Cameron and Trivedi (2022, 965–967) for additional discussion of these data types. See Wooldridge (2020, sec. 17.4) for an introduction to censored and truncated regression models.

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Regardless of the type of censoring, intreg requires the outcome to be stored in the dataset as interval data. That is, two dependent variables,  $depvar_1$  and  $depvar_2$ , are used to hold the endpoints of the interval. If the data are left-censored, the lower endpoint is  $-\infty$  and is represented by a missing value in  $depvar_1$ . If the data are right-censored, the upper endpoint is  $+\infty$  and is represented by a missing value in  $depvar_2$ . Point data are represented by the two endpoints being equal. Truly missing values of the dependent variable must be represented by missing values in both  $depvar_1$  and  $depvar_2$ .

## Example 1: Interval regression

womenwage2.dta contains the yearly wages of working women in interval form. Women were asked to indicate a category for their yearly income from employment. The categories were \$5,000 or less, \$5,001-\$10,000, ..., \$25,001-\$30,000, \$30,001-\$40,000, \$40,001-\$50,000, and more than \$50,000. The lower and upper endpoints of the wage categories (in \$1,000s) are recorded in variables wage1 and wage2. Below, we list the first 10 observations in wage1 and wage2.

. use https://www.stata-press.com/data/r18/womenwage2
(Wages of women, fictional data)

. list wage1 wage2 in 1/10

	wage1	wage2
1. 2. 3. 4. 5.	5 5 10 15	5 10 10 15 20
6. 7. 8. 9.	20 25 30 40 50	25 30 40 50

We see, for example, that the first respondent made \$5,000 or less in a year, that the second respondent made between \$5,001 and \$10,000 in a year, and so on. The tenth respondent made at least \$50,000 a year.

We now fit an interval regression model of women's wages using social and demographic characteristics as explanatory variables. The variables include the subject's age (age), years of schooling (school), job tenure (tenure), a dummy for living in a rural area (rural), and a dummy for never being married (nev\_mar).

. intreg wage1 wage2 age c.age#c.age i.nev\_mar i.rural school tenure

Fitting constant-only model:

Iteration 0: Log likelihood = -967.24956 Iteration 1: Log likelihood = -967.1368 Iteration 2: Log likelihood = -967.1368

Fitting full model:

Iteration 0: Log likelihood = -856.65324 Iteration 1: Log likelihood = -856.33294 Iteration 2: Log likelihood = -856.33293

Interval regression

Number of obs 488 Uncensored = 0 Left-censored = 14 Right-censored = 6 Interval-cens. = 468 LR chi2(6) = 221.61 = 0.0000

Prob > chi2

Log likelihood = -856.33293

	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
age	.7914438	.4433604	1.79	0.074	0775265	1.660414
c.age#c.age	0132624	.0073028	-1.82	0.069	0275757	.0010509
1.nev_mar 1.rural school	2075022 -3.043044 1.334721	.8119581 .7757324 .1357873	-0.26 -3.92 9.83	0.798 0.000 0.000	-1.798911 -4.563452 1.068583	1.383906 -1.522637 1.600859
tenure _cons	.8000664 -12.70238	.1045077	7.66 -1.99	0.000 0.000 0.046	.5952351 -25.1817	1.004898 2230583
/lnsigma	1.987823	.0346543	57.36	0.000	1.919902	2.055744
sigma	7.299626	.2529634			6.82029	7.81265

Because the conditional mean modeled by interval regression is linear, the coefficients are interpreted the same way they are in ordinary least-squares regression; see [R] regress. For example, residing in a rural area lowers the expected income by \$3,043 and each additional year of schooling raises the expected income by \$1,335.

Iteration 0:

/cut6

/cut7

/cut8

6.922726

7.34471

7.963441

.9215455

.9237628

.9338881

#### □ Technical note

Instead of using intervals to record wages, we could treat the outcome as categorical with a higher category corresponding to a higher wage. Here we fit an ordered probit model for wagecat, created based on groups defined by the intervals, by using oprobit (see [R] oprobit) with the same covariates:

. oprobit wagecat age c.age#c.age i.nev\_mar i.rural school tenure

Log likelihood = -881.1491

```
Log likelihood = -764.31729
Iteration 1:
Iteration 2:
              Log likelihood = -763.31191
Iteration 3: Log likelihood = -763.31049
Iteration 4: Log likelihood = -763.31049
                                                          Number of obs =
Ordered probit regression
                                                                              488
                                                          LR chi2(6)
                                                                        = 235.68
                                                                        = 0.0000
                                                          Prob > chi2
                                                          Pseudo R2
                                                                        = 0.1337
Log likelihood = -763.31049
               Coefficient Std. err.
                                                            [95% conf. interval]
     wagecat
                                            z
                                                  P>|z|
                  .1674519
                             .0620333
                                          2.70
                                                  0.007
                                                            .0458689
                                                                          .289035
 c.age#c.age
                -.0027983
                             .0010214
                                         -2.74
                                                  0.006
                                                           -.0048001
                                                                       -.0007964
                -.0046417
                                         -0.04
                                                  0.967
   1.nev_mar
                             .1126737
                                                            -.225478
                                                                         .2161946
                -.5270036
                             .1100449
                                         -4.79
                                                  0.000
                                                           -.7426875
                                                                       -.3113196
     1.rural
      school
                  .2010587
                             .0201189
                                          9.99
                                                  0.000
                                                            .1616263
                                                                         .2404911
                                          6.69
                                                  0.000
                  .0989916
                             .0147887
                                                            .0700063
                                                                          .127977
      tenure
       /cut1
                 2.650637
                                                                        4.406225
                             .8957245
                                                            .8950495
       /cut2
                 3.941018
                             .8979167
                                                            2.181134
                                                                        5.700903
                 5.085205
                                                                        6.860263
       /cut3
                             .9056582
                                                            3.310148
                 5.875534
                                                                        7.663204
       /cut4
                             .9120933
                                                            4.087864
       /cut5
                 6.468723
                              .918117
                                                            4.669247
                                                                        8.268199
```

We can directly compare the log likelihoods for the intreg and oprobit models because both likelihoods are discrete. If we had point data in our intreg estimation, the likelihood would be a mixture of discrete and continuous terms, and we could not compare it directly with the oprobit likelihood.

5.11653

5.534168

6.133054

8.728922

9.155252

9.793828

Here the oprobit log likelihood is significantly larger (that is, less negative), so it fits better than the intreg model. The intreg model assumes normality, but the distribution of wages is skewed and definitely nonnormal. Normality is more closely approximated if we model the log of wages.

```
. generate logwage1 = log(wage1)
(14 missing values generated)
. generate logwage2 = log(wage2)
(6 missing values generated)
. intreg logwage1 logwage2 age c.age#c.age i.nev_mar i.rural school tenure
Fitting constant-only model:
Iteration 0: Log likelihood = -889.23647
Iteration 1: Log likelihood = -889.06346
Iteration 2: Log likelihood = -889.06346
Fitting full model:
Iteration 0: Log likelihood = -773.81968
Iteration 1: Log likelihood = -773.36566
Iteration 2: Log likelihood = -773.36563
Interval regression
                                                    Number of obs
                                                           Uncensored =
                                                        Left-censored =
```

Log likelihood = -773.36563

= 0.0000

Prob > chi2

	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
age	.0645589	.0249954	2.58	0.010	.0155689	.1135489
c.age#c.age	0010812	.0004115	-2.63	0.009	0018878	0002746
1.nev_mar 1.rural school tenure	0058151 2098361 .0804832 .0397144	.0454867 .0439454 .0076783 .0058001	-0.13 -4.77 10.48 6.85	0.898 0.000 0.000 0.000	0949674 2959675 .0654341 .0283464	.0833371 1237047 .0955323 .0510825
_cons	.7084023	.3593193	1.97	0.049	.0041495	1.412655
/lnsigma	906989	.0356265	-25.46	0.000	9768157	8371623
sigma	.4037381	.0143838			.3765081	.4329373

The log likelihood of this intreg model is close to the oprobit log likelihood, and the z statistics for both models are similar.

### Stored results

intreg stores the following in e():

```
Scalars
    e(N)
                              number of observations
    e(N_unc)
                              number of uncensored observations
    e(N_lc)
                              number of left-censored observations
                              number of right-censored observations
    e(N_rc)
    e(N_int)
                              number of interval observations
                              number of parameters
    e(k)
                              number of auxiliary parameters
    e(k_aux)
    e(k_eq)
                              number of equations in e(b)
    e(k_eq_model)
                              number of equations in overall model test
                              number of dependent variables
    e(k_dv)
    e(df_m)
                              model degrees of freedom
    e(11)
                              log likelihood
    e(11_0)
                              log likelihood, constant-only model
    e(N_clust)
                              number of clusters
                              \chi^2
    e(chi2)
                              p-value for model \chi^2 test
    e(p)
    e(sigma)
                              standard error of sigma
    e(se_sigma)
                              rank of e(V)
    e(rank)
                              rank of e(V) for constant-only model
    e(rank0)
                              number of iterations
    e(ic)
    e(rc)
                              return code
    e(converged)
                              1 if converged, 0 otherwise
Macros
    e(cmd)
                              intreg
    e(cmdline)
                              command as typed
                              names of dependent variables
    e(depvar)
    e(wtype)
                              weight type
    e(wexp)
                              weight expression
    e(title)
                              title in estimation output
    e(clustvar)
                              name of cluster variable
                              linear offset variable
    e(offset)
    e(chi2type)
                              Wald or LR; type of model \chi^2 test
                              vcetype specified in vce()
    e(vce)
    e(vcetype)
                              title used to label Std. err.
    e(het)
                              heteroskedasticity, if het() specified
    e(ml_score)
                              program used to implement scores
    e(opt)
                              type of optimization
    e(which)
                              max or min; whether optimizer is to perform maximization or minimization
    e(ml_method)
                              type of ml method
    e(user)
                              name of likelihood-evaluator program
    e(technique)
                              maximization technique
    e(properties)
    e(predict)
                              program used to implement predict
    e(marginsok)
                              predictions allowed by margins
    e(asbalanced)
                              factor variables fvset as asbalanced
    e(asobserved)
                              factor variables fyset as asobserved
Matrices
                              coefficient vector
    e(b)
    e(Cns)
                              constraints matrix
    e(ilog)
                              iteration log (up to 20 iterations)
                              gradient vector
    e(gradient)
                              variance-covariance matrix of the estimators
    e(V)
    e(V_modelbased)
                              model-based variance
Functions
    e(sample)
                              marks estimation sample
```

In addition to the above, the following is stored in r():

Matrices

r(table)

matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

### Methods and formulas

The regression equation of interest is

$$y_j = \mathbf{x}_j \boldsymbol{\beta} + \epsilon_j$$

where  $y_j$  is a continuous outcome for the jth observation—either observed or unobserved—with covariates  $\mathbf{x}_j$  and corresponding coefficients  $\boldsymbol{\beta}$ . The model assumes that the error term is normally distributed;  $\epsilon \sim N(0, \sigma^2)$ .

For observations  $j \in \mathcal{C}$ , we observe  $y_j$ , that is, point data. Observations  $j \in \mathcal{I}$  are intervals; we know only that the unobserved  $y_j$  is in the interval  $[y_{1j}, y_{2j}]$ . For these observations, the likelihood contribution is  $\Pr(y_{1j} \leq Y_j \leq y_{2j})$ , where  $Y_j$  denotes the random variable representing the dependent variable in the model. Observations  $j \in \mathcal{L}$  are left-censored; we know only that the unobserved  $y_j$  is less than or equal to  $y_{\mathcal{L}j}$ , a censoring value that we do know. Similarly, observations  $j \in \mathcal{R}$  are right-censored; we know only that the unobserved  $y_j$  is greater than or equal to  $y_{\mathcal{R}j}$ . The likelihoods for these censored observations contain terms of the form  $\Pr(Y_j \leq y_{\mathcal{L}j})$  for left-censored data and  $\Pr(Y_j \geq y_{\mathcal{R}j})$  for right-censored data.

The log likelihood is

$$\begin{split} \ln L &= -\frac{1}{2} \sum_{j \in \mathcal{C}} w_j \left\{ \left( \frac{y_j - \mathbf{x}_j \boldsymbol{\beta}}{\sigma} \right)^2 + \log 2\pi \sigma^2 \right\} \\ &+ \sum_{j \in \mathcal{L}} w_j \log \Phi \left( \frac{y_{\mathcal{L}j} - \mathbf{x}_j \boldsymbol{\beta}}{\sigma} \right) \\ &+ \sum_{j \in \mathcal{R}} w_j \log \left\{ 1 - \Phi \left( \frac{y_{\mathcal{R}j} - \mathbf{x}_j \boldsymbol{\beta}}{\sigma} \right) \right\} \\ &+ \sum_{j \in \mathcal{I}} w_j \log \left\{ \Phi \left( \frac{y_{2j} - \mathbf{x}_j \boldsymbol{\beta}}{\sigma} \right) - \Phi \left( \frac{y_{1j} - \mathbf{x}_j \boldsymbol{\beta}}{\sigma} \right) \right\} \end{split}$$

where  $\Phi()$  is the cumulative standard normal distribution and  $w_j$  is the weight for the jth observation. If no weights are specified,  $w_j = 1$ . If aweights are specified,  $w_j = 1$ , and  $\sigma$  is replaced by  $\sigma/\sqrt{a_j}$  in the above, where  $a_j$  are the aweights normalized to sum to N.

When the het() option is specified,  $\sigma$  is modeled as  $ln(\sigma) = z'_j \gamma$ , where z represents the variables in het() and  $\gamma$  is a vector of the estimated parameters to model the variance.

Note that the likelihood for intreg subsumes that of the tobit models; see [R] tobit.

Maximization is as described in [R] Maximize. intreg stores the estimated  $\sigma$  in e(b) in the log metric; therefore, if you want to provide an initial value for  $\sigma$  or to specify a constraint on it, ensure you do so on the log scale.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster clustvar), respectively. See [P] \_robust, particularly Maximum likelihood estimators and Methods and formulas.

intreg also supports estimation with survey data. For details on VCEs with survey data, see [SVY] Variance estimation.

#### References

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## Also see

[R] **intreg postestimation** — Postestimation tools for intreg

[R] **regress** — Linear regression

[R] **tobit** — Tobit regression

[BAYES] bayes: intreg — Bayesian interval regression

[ERM] eintreg — Extended interval regression

[FMM] **fmm: intreg** — Finite mixtures of interval regression models

[ME] meintreg — Multilevel mixed-effects interval regression

[ST] **stintreg** — Parametric models for interval-censored survival-time data

[SVY] **svy estimation** — Estimation commands for survey data

[XT] **xtintreg** — Random-effects interval-data regression models

[XT] **xttobit** — Random-effects tobit models

[U] 20 Estimation and postestimation commands

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