# Title

**pinv( )** — Moore–Penrose pseudoinverse

Description    Syntax    Remarks and examples    Conformability
Diagnostics    References    Also see

## Description

pinv($A$) returns the unique Moore–Penrose pseudoinverse of real or complex, symmetric or non-symmetric, square or nonsquare matrix $A$.

pinv($A$, *rank*) does the same thing, and it returns in *rank* the rank of $A$.

pinv($A$, *rank*, *tol*) does the same thing, and it allows you to specify the tolerance used to determine the rank of $A$, which is also used in the calculation of the pseudoinverse. See [M-5] **svsolve( )** and [M-1] **Tolerance** for information on the optional *tol* argument.

_pinv($A$) and _pinv($A$, *tol*) do the same thing as pinv(), except that $A$ is replaced with its inverse and the rank is returned.

## Syntax

> *numeric matrix*     pinv(*numeric matrix A*)
>
> *numeric matrix*     pinv(*numeric matrix A*, *rank*)
>
> *numeric matrix*     pinv(*numeric matrix A*, *rank*, *real scalar tol*)
>
> *real scalar*     _pinv(*numeric matrix A*)
>
> *real scalar*     _pinv(*numeric matrix A*, *real scalar tol*)

where the type of *rank* is irrelevant; the rank of $A$ is returned there.

To obtain a generalized inverse of a symmetric matrix with a different normalization, see [M-5] **invsym( )**.

## Remarks and examples

The Moore–Penrose pseudoinverse is also known as the Moore–Penrose inverse and as the generalized inverse. Whatever you call it, the pseudoinverse $A*$ of $A$ satisfies four conditions,

$$A(A*)A = A$$
$$(A*)A(A*) = A*$$
$$(A(A*))' = A(A*)$$
$$((A*)A)' = (A*)A$$

where the transpose operator $'$ is understood to mean the conjugate transpose when $A$ is complex. Also, if $A$ is of full rank, then

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$$A* = A^{-1}$$

pinv($A$) is logically equivalent to svsolve($A$, I(rows($A$))); see [M-5] **svsolve( )** for details and for use of the optional *tol* argument.

## Conformability

pinv($A$, *rank*, *tol*):

> *input*:

|  |  |  |  |
|---|---|---|---|
| $A$: | $r \times c$ | |
| *tol*: | $1 \times 1$ | (optional) |

> *output*:

|  |  |  |
|---|---|---|
| *rank*: | $1 \times 1$ | (optional) |
| *result*: | $c \times r$ | |

_pinv($A$, *tol*):

> *input*:

|  |  |  |
|---|---|---|
| $A$: | $r \times c$ | |
| *tol*: | $1 \times 1$ | (optional) |

> *output*:

|  |  |  |
|---|---|---|
| $A$: | $c \times r$ | |
| *result*: | $1 \times 1$ | (containing rank) |

## Diagnostics

The inverse returned by these functions is real if $A$ is real and is complex if $A$ is complex.

pinv($A$, *rank*, *tol*) and _pinv($A$, *tol*) return missing results if $A$ contains missing values.

pinv() and _pinv() also return missing values if the algorithm for computing the SVD, [M-5] **svd( )**, fails to converge. This is a near zero-probability event. Here *rank* also is returned as missing.

See [M-5] **svsolve( )** and [M-1] **Tolerance** for information on the optional *tol* argument.

## References

James, I. M. 2002. *Remarkable Mathematicians: From Euler to von Neumann*. Cambridge: Cambridge University Press.

Moore, E. H. 1920. On the reciprocal of the general algebraic matrix. *Bulletin of the American Mathematical Society* 26: 394–395.

Penrose, R. 1955. A generalized inverse for matrices. *Mathematical Proceedings of the Cambridge Philosophical Society* 51: 406–413. https://doi.org/10.1017/S0305004100030401.

# Also see

Eliakim Hastings Moore (1862–1932) was born in Marietta, Ohio. He studied mathematics and astronomy at Yale and was awarded a PhD for a thesis on $n$-dimensional geometry. After a year studying in Germany and teaching posts at Northwestern and Yale, he settled at the University of Chicago in 1892. Moore worked on algebra, including fields and groups, the foundations of geometry and the foundations of analysis, algebraic geometry, number theory, and integral equations. He was an inspiring teacher and a great organizer in American mathematics, playing an important part in the early years of the American Mathematical Society.

Roger Penrose (1931– ) was born in Colchester in England. His father was a statistically minded medical geneticist and his mother was a doctor. Penrose studied mathematics at University College London and Cambridge and published an article on generalized matrix inverses in 1955. He taught and researched at several universities in Great Britain and the United States before being appointed Rouse Ball Professor of Mathematics at Oxford in 1973. Penrose is celebrated for research contributions ranging from cosmology and general relativity (including work with Stephen Hawking) to pure mathematics (including results on tilings of the plane) and for semipopular and wide-ranging books making controversial connections between physics, computers, mind, and consciousness. He was knighted in 1994 and jointly awarded the Nobel Prize for Physics in 2020.