Title

Hilbert( ) — Hilbert matrices

Description

`Hilbert(`*n*`)` returns the $n \times n$ Hilbert matrix, defined as matrix H with elements $H_{ij} = 1/(i+j-1)$.

`invHilbert(`*n*`)` returns the inverse of the $n \times n$ Hilbert matrix, defined as the matrix with elements $-1^{i+j}\ (i+j-1) \times \text{comb}(n+i-1, n-j) \times \text{comb}(n+j-1, n-i) \times \text{comb}(i+j-2, i-1)^2$.

Syntax

real matrix `Hilbert(`*real scalar n*`)`

real matrix `invHilbert(`*real scalar n*`)`

Remarks and examples

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`Hilbert(`*n*`)` and `invHilbert(`*n*`)` are used in testing Mata. Hilbert matrices are notoriously ill conditioned. The determinants of the first five Hilbert matrices are 1, 1/12, 1/2,160, 1/6,048,000, and 1/266,716,800,000.

Conformability

`Hilbert(`*n*`)`, `invHilbert(`*n*`)`:

n:	1×1
result:	$n \times n$

Diagnostics

If n is not an integer, trunc(n) is used.

David Hilbert (1862–1943) was born near Königsberg, Prussia (now Kaliningrad, Russia), and studied mathematics at the university there. He joined the staff from 1886 to 1895, when he moved to Göttingen, where he stayed despite tempting offers to move. Hilbert was one of the outstanding mathematicians of his time, producing major work in several fields, including invariant theory, algebraic number theory, the foundations of geometry, functional analysis, integral equations, and the calculus of variations. In 1900 he identified 23 key problems in an address to the Second International Congress of Mathematicians in Paris that continues to influence directions in research (Hilbert 1902). Hilbert's most noteworthy contribution is the concept of a Hilbert space. His work on what are now known as Hilbert matrices was published in 1894.

References

Choi, M.-D. 1983. Tricks or treats with the Hilbert matrix. *American Mathematical Monthly* 90: 301–312. https://doi.org/10.2307/2975779.

Hilbert, D. 1894. Ein Beitrag zur Theorie des Legendreschen Polynoms. *Acta Mathematica* 18: 155–159. https://doi.org/10.1007/BF02418278.

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Reid, C. 1970. *Hilbert*. Berlin: Springer.

Also see

[M-4] **Standard** — Functions to create standard matrices