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# Statistical functions

Contents Functions References Also see

# **Contents**

betaden(a,b,x)	the probability density of the beta distribution, where $a$ and $b$ are
	the shape parameters; 0 if $x < 0$ or $x > 1$
$\texttt{binomial}(n, k, \theta)$	the probability of observing $floor(k)$ or fewer successes in $floor(n)$ trials when the probability of a success on one trial is $\theta$ ; 0 if $k < 0$ ; or 1 if $k > n$
binomialp(n, k, p)	the probability of observing $floor(k)$ successes in $floor(n)$ trials when the probability of a success on one trial is $p$
$\texttt{binomialtail}(n,k,\theta)$	the probability of observing $floor(k)$ or more successes in $floor(n)$ trials when the probability of a success on one trial is $\theta$ ; 1 if $k < 0$ ; or 0 if $k > n$
$\texttt{binormal}(h,k,\rho)$	the joint cumulative distribution $\Phi(h,k,\rho)$ of bivariate normal with correlation $\rho$
$\operatorname{cauchy}(a,b,x)$	the cumulative Cauchy distribution with location parameter $\boldsymbol{a}$ and scale parameter $\boldsymbol{b}$
$\operatorname{cauchyden}(a,b,x)$	the probability density of the Cauchy distribution with location parameter $\boldsymbol{a}$ and scale parameter $\boldsymbol{b}$
cauchytail(a,b,x)	the reverse cumulative (upper tail or survivor) Cauchy distribution with location parameter $\boldsymbol{a}$ and scale parameter $\boldsymbol{b}$
chi2(df,x)	the cumulative $\chi^2$ distribution with $d\!f$ degrees of freedom; 0 if $x<0$
$\mathtt{chi2den}(df,x)$	the probability density of the $\chi^2$ distribution with $d\!f$ degrees of freedom; 0 if $x<0$
chi2tail(df,x)	the reverse cumulative (upper tail or survivor) $\chi^2$ distribution with $df$ degrees of freedom; 1 if $x<0$
dgammapda(a,x)	$rac{\partial P(a,x)}{\partial a}$ , where $P(a,x)=$ gammap( $a$ , $x$ ); 0 if $x<0$
dgammapdada(a,x)	$\frac{\partial^2 P(a,x)}{\partial a^2}$ , where $P(a,x)=\mathrm{gammap}(a,x)$ ; 0 if $x<0$
$\operatorname{dgammapdadx}(a,x)$	$rac{\partial^2 P(a,x)}{\partial a \partial x}$ , where $P(a,x) = \mathtt{gammap}(a,x)$ ; 0 if $x < 0$
$\operatorname{dgammapdx}(a,x)$	$rac{\partial P(a,x)}{\partial x}$ , where $P(a,x)=$ gammap( $a$ , $x$ ); 0 if $x<0$
dgammapdxdx(a,x)	$\frac{\partial^2 P(a,x)}{\partial x^2}$ , where $P(a,x)=$ gammap( $a$ , $x$ ); 0 if $x<0$
dunnettprob(k, df, x)	the cumulative multiple range distribution that is used in Dunnett's multiple-comparison method with $k$ ranges and $d\!f$ degrees of freedom; 0 if $x<0$
exponential(b,x)	the cumulative exponential distribution with scale $b$
exponentialden(b,x)	the probability density function of the exponential distribution with scale $\boldsymbol{b}$
exponentialtail(b,x)	the reverse cumulative exponential distribution with scale $\boldsymbol{b}$

$F(df_1, df_2, f)$	the cumulative $F$ distribution with $df_1$ numerator and $df_2$ denominator degrees of freedom: $\mathbf{F}(df_1,df_2,f)=\int_0^f\mathbf{Fden}(df_1,df_2,t)$ $dt;$ 0 if $f<0$
$\texttt{Fden}(df_1, df_2, f)$	the probability density function of the $F$ distribution with $df_1$ numerator and $df_2$ denominator degrees of freedom; 0 if $f<0$
$\texttt{Ftail}(df_1, df_2, f)$	the reverse cumulative (upper tail or survivor) $F$ distribution with $df_1$ numerator and $df_2$ denominator degrees of freedom; 1 if $f<0$
gammaden(a,b,g,x)	the probability density function of the gamma distribution; 0 if $x < g $
gammap(a,x)	the cumulative gamma distribution with shape parameter $a; \ {\rm 0} \ \ {\rm if} \ \ x < 0$
gammaptail(a,x)	the reverse cumulative (upper tail or survivor) gamma distribution with shape parameter $a;1$ if $x<0$
$\verb hypergeometric (N,K,n,k) $	the cumulative probability of the hypergeometric distribution
${\tt hypergeometricp}(N,K,n,k)$	the hypergeometric probability of $k$ successes out of a sample of size $n$ , from a population of size $N$ containing $K$ elements that have the attribute of interest
ibeta(a,b,x)	the cumulative beta distribution with shape parameters $a$ and $b$ ; 0 if $x<0$ ; or 1 if $x>1$
ibetatail(a,b,x)	the reverse cumulative (upper tail or survivor) beta distribution with shape parameters $a$ and $b$ ; 1 if $x<0$ ; or 0 if $x>1$
igaussian(m,a,x)	the cumulative inverse Gaussian distribution with mean $m$ and shape parameter $a; \ {\rm 0} \ \ {\rm if} \ x \leq 0$
igaussianden(m,a,x)	the probability density of the inverse Gaussian distribution with mean $m$ and shape parameter $a;$ 0 if $x\leq 0$
igaussiantail(m,a,x)	the reverse cumulative (upper tail or survivor) inverse Gaussian distribution with mean $m$ and shape parameter $a;$ 1 if $x \leq 0$
invbinomial(n,k,p)	the inverse of the cumulative binomial; that is, $\theta$ ( $\theta$ = probability of success on one trial) such that the probability of observing floor( $k$ ) or fewer successes in floor( $n$ ) trials is $p$
invbinomialtail(n,k,p)	the inverse of the right cumulative binomial; that is, $\theta$ ( $\theta$ = probability of success on one trial) such that the probability of observing floor( $k$ ) or more successes in floor( $n$ ) trials is $p$
invcauchy(a,b,p)	the inverse of cauchy(): if cauchy( $a,b,x$ ) = $p$ , then invcauchy( $a,b,p$ ) = $x$
invcauchytail(a,b,p)	the inverse of cauchytail(): if cauchytail( $a,b,x$ ) = $p$ , then invcauchytail( $a,b,p$ ) = $x$
invchi2(df,p)	the inverse of chi2(): if chi2( $df$ , $x$ ) = $p$ , then invchi2( $df$ , $p$ ) = $x$
${\tt invchi2tail}(df,p)$	the inverse of chi2tail(): if chi2tail( $df$ , $x$ ) = $p$ , then invchi2tail( $df$ , $p$ ) = $x$
invdunnettprob(k, df, p)	the inverse cumulative multiple range distribution that is used in Dunnett's multiple-comparison method with $k$ ranges and $df$ degrees of freedom
invexponential(b,p)	the inverse cumulative exponential distribution with scale $b$ : if exponential( $b$ , $x$ ) = $p$ , then invexponential( $b$ , $p$ ) = $x$

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invexponential tail(b, p)
                               the inverse reverse cumulative exponential distribution with scale b:
                                   if exponentialtail(b, x) = p, then
                                   invexponentialtail(b, p) = x
                               the inverse cumulative F distribution: if F(df_1, df_2, f) = p, then
invF(df_1, df_2, p)
                                   invF(df_1, df_2, p) = f
invFtail(df_1, df_2, p)
                               the inverse reverse cumulative (upper tail or survivor) F distribution:
                                  if Ftail(df_1, df_2, f) = p, then invFtail(df_1, df_2, p) = f
                               the inverse cumulative gamma distribution: if gammap(a, x) = p,
invgammap(a, p)
                                   then invgammap(a, p) = x
                               the inverse reverse cumulative (upper tail or survivor) gamma distri-
invgammaptail(a, p)
                                   bution: if gammaptail (a, x) = p, then invgammaptail (a, p)
                               the inverse cumulative beta distribution: if ibeta(a,b,x) = p,
invibeta(a,b,p)
                                  then invibeta(a,b,p)=x
invibetatail(a,b,p)
                               the inverse reverse cumulative (upper tail or survivor) beta distribu-
                                   tion: if ibetatail(a,b,x) = p, then invibetatail(a,b,p)
                               the inverse of igaussian(): if
invigaussian(m,a,p)
                                   igaussian(m,a,x) = p, then invigaussian(m,a,p) = x
invigaussiantail(m, a, p)
                               the inverse of igaussiantail(): if
                                   igaussiantail(m,a,x)=p, then
                                   invigaussiantail(m, a, p) = x
                               the inverse of laplace(): if laplace(m,b,x) = p, then
invlaplace(m,b,p)
                                   invlaplace(m,b,p) = x
                               the inverse of laplacetail(): if laplacetail(m, b, x) = p,
invlaplacetail(m,b,p)
                                   then invlaplacetail (m, b, p) = x
invlogistic(p)
                               the inverse cumulative logistic distribution: if logistic(x) = p,
                                   then invlogistic(p) = x
                               the inverse cumulative logistic distribution: if logistic(s, x) = p,
invlogistic(s,p)
                                   then invlogistic(s,p) = x
                               the inverse cumulative logistic distribution: if logistic(m, s, x)
invlogistic(m,s,p)
                                   = p, then invlogistic(m, s, p) = x
invlogistictail(p)
                               the inverse reverse cumulative logistic distribution: if
                                   logistictail(x) = p, then invlogistictail(p) = x
                               the inverse reverse cumulative logistic distribution: if
invlogistictail(s,p)
                                   logistictail(s,x) = p, then invlogistictail(s,p) = x
invlogistictail(m,s,p)
                               the inverse reverse cumulative logistic distribution: if
                                   logistictail(m, s, x) = p, then
                                   invlogistictail(m,s,p) = x
                               the value of the negative binomial parameter, p, such that q =
invnbinomial (n, k, q)
                                  nbinomial(n,k,p)
                               the value of the negative binomial parameter, p, such that
invnbinomialtail(n, k, q)
                                   q = nbinomialtail(n, k, p)
                               the inverse cumulative noncentral \chi^2 distribution: if {\tt nchi2}(df , np , x)=p, then {\tt invnchi2}(df , np , p)=x
invnchi2(df, np, p)
                               the inverse reverse cumulative (upper tail or survivor) non-
invnchi2tail(df, np, p)
                                  central \chi^2 distribution: if nchi2tail(df, np, x) = p, then
                                   invnchi2tail(df, np, p) = x
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invnF(df_1, df_2, np, p)
                               the inverse cumulative noncentral F distribution: if
                                  nF(df_1, df_2, np, f) = p, then invnF(df_1, df_2, np, p) = f
invnFtail(df_1, df_2, np, p)
                               the inverse reverse cumulative (upper tail or survivor) noncen-
                                  tral F distribution: if nFtail(df_1, df_2, np, f) = p, then
                                  invnFtail(df_1, df_2, np, p) = f
                               the inverse cumulative noncentral beta distribution: if
invnibeta(a,b,np,p)
                                  nibeta(a,b,np,x) = p, then invibeta(a,b,np,p) = x
                               the inverse cumulative standard normal distribution: if normal(z)
invnormal(p)
                                  = p, then invnormal(p) = z
invnt(df, np, p)
                               the inverse cumulative noncentral Student's t distribution: if
                                  nt(df, np, t) = p, then invnt(df, np, p) = t
                               the inverse reverse cumulative (upper tail or survivor) noncen-
invnttail(df, np, p)
                                  tral Student's t distribution: if nttail(df, np, t) = p, then
                                  invnttail(df, np, p) = t
                               the Poisson mean such that the cumulative Poisson distribution eval-
invpoisson(k,p)
                                  uated at k is p: if poisson(m,k) = p, then invpoisson(k,p)
                               the Poisson mean such that the reverse cumulative Poisson distri-
invpoissontail(k,q)
                                  bution evaluated at k is q: if poissontail(m,k) = q, then
                                  invpoissontail(k,q) = m
                               the inverse cumulative Student's t distribution: if t(df,t) = p,
invt(df, p)
                                  then invt(df, p) = t
                               the inverse reverse cumulative (upper tail or survivor) Student's
invttail(df, p)
                                  t distribution: if ttail(df, t) = p, then invttail(df, p) = t
                               the inverse cumulative Tukey's Studentized range distribution with
invtukeyprob(k, df, p)
                                  k ranges and df degrees of freedom
                               the inverse cumulative Weibull distribution with shape a and scale
invweibull(a,b,p)
                                  b: if weibull(a,b,x) = p, then invweibull(a,b,p) = x
invweibull(a,b,g,p)
                               the inverse cumulative Weibull distribution with shape a, scale b,
                                  and location g: if weibull(a,b,g,x) = p, then
                                  invweibull(a,b,g,p) = x
                               the inverse cumulative Weibull (proportional hazards) distribution
invweibullph(a,b,p)
                                  with shape a and scale b: if weibullph(a,b,x) = p, then
                                  invweibullph(a,b,p) = x
                               the inverse cumulative Weibull (proportional hazards) distribution
invweibullph(a,b,g,p)
                                  with shape a, scale b, and location g: if weibullph(a,b,g,x) =
                                  p, then invweibullph(a,b,g,p) = x
                               the inverse reverse cumulative Weibull (proportional hazards) distri-
invweibullphtail(a,b,p)
                                  bution with shape a and scale b: if weibullphtail(a,b,x) = p,
                                  then invweibullphtail(a,b,p) = x
                               the inverse reverse cumulative Weibull (proportional hazards)
invweibullphtail(a,b,g,p)
                                  distribution with shape a, scale b, and location g: if
                                  weibullphtail(a,b,g,x) = p, then
                                  invweibullphtail(a,b,g,p) = x
invweibulltail(a, b, p)
                               the inverse reverse cumulative Weibull distribution with shape a and
                                  scale b: if weibulltail(a,b,x) = p, then
                                  invweibulltail(a,b,p) = x
                               the inverse reverse cumulative Weibull distribution with shape a,
invweibulltail(a, b, q, p)
                                  scale b, and location g: if weibulltail(a,b,g,x) = p, then
                                  invweibulltail(a,b,g,p) = x
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```
laplace(m,b,x)
laplaceden(m,b,x)
laplacetail(m,b,x)
lncauchyden(a,b,x)
lnigammaden(a,b,x)
lnigaussianden(m,a,x)
lniwishartden(df, V, X)
lnlaplaceden(m,b,x)
lnmvnormalden(M, V, X)
lnnormal(z)
lnnormalden(z)
lnnormalden(x,\sigma)
lnnormalden(x, \mu, \sigma)
lnwishartden(df, V, X)
logistic(x)
logistic(s,x)
logistic(m,s,x)
logisticden(x)
logisticden(s,x)
logisticden(m,s,x)
logistictail(x)
logistictail(s,x)
logistictail(m,s,x)
nbetaden(a,b,np,x)
nbinomial(n,k,p)
```

Statistical functions the cumulative Laplace distribution with mean m and scale paramthe probability density of the Laplace distribution with mean m and scale parameter b the reverse cumulative (upper tail or survivor) Laplace distribution with mean m and scale parameter bthe natural logarithm of the density of the Cauchy distribution with location parameter a and scale parameter b the natural logarithm of the inverse gamma density, where a is the shape parameter and b is the scale parameter the natural logarithm of the inverse Gaussian density with mean mand shape parameter a the natural logarithm of the density of the inverse Wishart distribution; missing if df < n-1the natural logarithm of the density of the Laplace distribution with mean m and scale parameter bthe natural logarithm of the multivariate normal density the natural logarithm of the cumulative standard normal distribution the natural logarithm of the standard normal density, N(0,1)the natural logarithm of the normal density with mean 0 and standard deviation  $\sigma$ the natural logarithm of the normal density with mean  $\mu$  and standard deviation  $\sigma$ ,  $N(\mu, \sigma^2)$ the natural logarithm of the density of the Wishart distribution; missing if df < n-1the cumulative logistic distribution with mean 0 and standard deviation  $\pi/\sqrt{3}$ deviation  $s\pi/\sqrt{3}$ the cumulative logistic distribution with mean m, scale s, and standard deviation  $s\pi/\sqrt{3}$ 

the cumulative logistic distribution with mean 0, scale s, and standard

the density of the logistic distribution with mean 0 and standard deviation  $\pi/\sqrt{3}$ 

the density of the logistic distribution with mean 0, scale s, and standard deviation  $s\pi/\sqrt{3}$ 

the density of the logistic distribution with mean m, scale s, and standard deviation  $s\pi/\sqrt{3}$ 

the reverse cumulative logistic distribution with mean 0 and standard deviation  $\pi/\sqrt{3}$ 

the reverse cumulative logistic distribution with mean 0, scale s, and standard deviation  $s\pi/\sqrt{3}$ 

the reverse cumulative logistic distribution with mean m, scale s, and standard deviation  $s\pi/\sqrt{3}$ 

the probability density function of the noncentral beta distribution; 0 if x < 0 or x > 1

the cumulative probability of the negative binomial distribution

nbinomialp(n,k,p)

```
nbinomialtail(n, k, p)
                                 the reverse cumulative probability of the negative binomial distri-
                                 the cumulative noncentral \chi^2 distribution; 0 if x<0
nchi2(df, np, x)
                                 the probability density of the noncentral \chi^2 distribution; 0 if x<0
nchi2den(df, np, x)
                                 the reverse cumulative (upper tail or survivor) noncentral \chi^2 distri-
nchi2tail(df, np, x)
                                    bution; 1 if x < 0
nF(df_1, df_2, np, f)
                                 the cumulative noncentral F distribution with df_1 numerator and
                                     df_2 denominator degrees of freedom and noncentrality parameter
                                    np; 0 if f < 0
nFden(df_1, df_2, np, f)
                                 the probability density function of the noncentral F distribution
                                     with df_1 numerator and df_2 denominator degrees of freedom and
                                     noncentrality parameter np; 0 if f < 0
nFtail(df_1, df_2, np, f)
                                 the reverse cumulative (upper tail or survivor) noncentral F dis-
                                    tribution with df_1 numerator and df_2 denominator degrees of
                                    freedom and noncentrality parameter np; 1 if f < 0
nibeta(a,b,np,x)
                                 the cumulative noncentral beta distribution; 0 if x < 0; or 1 if
                                    x > 1
normal(z)
                                 the cumulative standard normal distribution
normalden(z)
                                 the standard normal density, N(0,1)
normalden(x, \sigma)
                                 the normal density with mean 0 and standard deviation \sigma
normalden(x, \mu, \sigma)
                                 the normal density with mean \mu and standard deviation \sigma, N(\mu, \sigma^2)
                                 the noncentrality parameter, np, for noncentral \chi^2: if
npnchi2(df,x,p)
                                    nchi2(df, np, x) = p, then npnchi2(df, x, p) = np
npnF(df_1, df_2, f, p)
                                 the noncentrality parameter, np, for the noncentral F: if
                                    nF(df_1, df_2, np, f) = p, then npnF(df_1, df_2, f, p) = np
                                 the noncentrality parameter, np, for the noncentral Student's
npnt(df,t,p)
                                     t distribution: if nt(df, np, t) = p, then npnt(df, t, p) = np
nt(df, np, t)
                                 the cumulative noncentral Student's t distribution with df degrees
                                    of freedom and noncentrality parameter np
ntden(df, np, t)
                                 the probability density function of the noncentral Student's
                                     t distribution with df degrees of freedom and noncentrality pa-
                                    rameter np
nttail(df, np, t)
                                 the reverse cumulative (upper tail or survivor) noncentral Student's
                                     t distribution with df degrees of freedom and noncentrality pa-
                                    rameter np
poisson(m,k)
                                 the probability of observing floor(k) or fewer outcomes that are
                                    distributed as Poisson with mean m
                                 the probability of observing floor(k) outcomes that are distributed
poissonp(m,k)
                                     as Poisson with mean m
                                 the probability of observing floor(k) or more outcomes that are
poissontail(m,k)
                                     distributed as Poisson with mean m
t(df,t)
                                 the cumulative Student's t distribution with df degrees of freedom
tden(df,t)
                                 the probability density function of Student's t distribution
ttail(df,t)
                                 the reverse cumulative (upper tail or survivor) Student's t distribution;
                                    the probability T > t
                                 the cumulative Tukey's Studentized range distribution with k ranges
tukeyprob(k, df, x)
                                     and df degrees of freedom; 0 if x < 0
```

the negative binomial probability

```
weibull(a,b,x)
weibull(a,b,g,x)
weibullden(a,b,g,x)
weibullden(a,b,g,x)
weibullph(a,b,x)
weibullph(a,b,g,x)
weibullphden(a,b,g,x)
weibullphden(a,b,g,x)
weibullphtail(a,b,g,x)
weibullphtail(a,b,g,x)
```

weibulltail(a,b,q,x)

- the cumulative Weibull distribution with shape a and scale b
- the cumulative Weibull distribution with shape a, scale b, and location q
- the probability density function of the Weibull distribution with shape a and scale b
- the probability density function of the Weibull distribution with shape a, scale b, and location g
- the cumulative Weibull (proportional hazards) distribution with shape a and scale b
- the cumulative Weibull (proportional hazards) distribution with shape a, scale b, and location g
- the probability density function of the Weibull (proportional hazards) distribution with shape a and scale b
- the probability density function of the Weibull (proportional hazards) distribution with shape a, scale b, and location g
- the reverse cumulative Weibull (proportional hazards) distribution with shape  $\boldsymbol{a}$  and scale  $\boldsymbol{b}$
- the reverse cumulative Weibull (proportional hazards) distribution with shape a, scale b, and location g
- the reverse cumulative Weibull distribution with shape  $\boldsymbol{a}$  and scale  $\boldsymbol{b}$
- the reverse cumulative Weibull distribution with shape a, scale b, and location g

# **Functions**

Statistical functions are listed alphabetically under the following headings:

Beta and noncentral beta distributions

Binomial distribution

Cauchy distribution

 $\chi^2$  and noncentral  $\chi^2$  distributions

Dunnett's multiple range distribution

Exponential distribution

F and noncentral F distributions

Gamma distribution

Hypergeometric distribution

Inverse Gaussian distribution

Laplace distribution

Logistic distribution

Negative binomial distribution

Normal (Gaussian), binormal, and multivariate normal distributions

Poisson distribution

Student's t and noncentral Student's t distributions

Tukey's Studentized range distribution

Weibull distribution

Weibull (proportional hazards) distribution

Wishart distribution

#### Beta and noncentral beta distributions

betaden(a,b,x)

Description: the probability density of the beta distribution, where a and b are the shape parameters;

0 if x < 0 or x > 1

The probability density of the beta distribution is

$$\mathtt{betaden}(a,b,x) = \frac{x^{a-1}(1-x)^{b-1}}{\int_0^\infty t^{a-1}(1-t)^{b-1}dt} = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$$

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307

Domain x: -8e+307 to 8e+307; interesting domain is  $0 \le x \le 1$ 

Range: 0 to 8e+307

#### ibeta(a,b,x)

Description: the cumulative beta distribution with shape parameters a and b; 0 if x < 0; or 1 if x > 1The cumulative beta distribution with shape parameters a and b is defined by

$$I_x(a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1} (1-t)^{b-1} dt$$

ibeta() returns the regularized incomplete beta function, also known as the incomplete beta function ratio. The incomplete beta function without regularization is given by (gamma(a)\*gamma(b)/gamma(a+b))\*ibeta(a,b,x) or, better when a or b might be large,  $\exp(\operatorname{lngamma}(a) + \operatorname{lngamma}(b) - \operatorname{lngamma}(a+b)) * ibeta(a, b, x).$ 

Here is an example of the use of the regularized incomplete beta function. Although Stata has a cumulative binomial function (see binomial()), the probability that an event occurs k or fewer times in n trials, when the probability of one event is p, can be evaluated as cond (k==n,1,1-ibeta(k+1,n-k,p)). The reverse cumulative binomial (the probability that an event occurs k or more times) can be evaluated as cond(k==0,1,ibeta(k,n-k+1,p)). See Press et al. (2007, 270-273) for a more complete description and for suggested uses for this function.

Domain a: 1e-10 to 1e+17Domain b: 1e-10 to 1e+17

Domain x: -8e+307 to 8e+307; interesting domain is  $0 \le x \le 1$ 

Range: 0 to 1

#### ibetatail(a,b,x)

Description: the reverse cumulative (upper tail or survivor) beta distribution with shape parameters a and b; 1 if x < 0; or 0 if x > 1

> The reverse cumulative (upper tail or survivor) beta distribution with shape parameters a and b is defined by

$$ibetatail(a,b,x) = 1 - ibeta(a,b,x) = \int_x^1 betaden(a,b,t) dt$$

ibetatail() is also known as the complement to the incomplete beta function (ratio).

Domain a: 1e-10 to 1e+17 Domain b: 1e-10 to 1e+17

Domain x: -8e+307 to 8e+307; interesting domain is  $0 \le x \le 1$ 

Range: 0 to 1

### invibeta(a,b,p)

Description: the inverse cumulative beta distribution: if ibeta(a,b,x) = p,

then invibeta(a,b,p) = x

Domain a: 1e-10 to 1e+17 Domain b: 1e-10 to 1e+17

Domain p: 0 to 1 Range: 0 to 1

# invibetatail(a,b,p)

Description: the inverse reverse cumulative (upper tail or survivor) beta distribution: if

ibetatail(a,b,x) = p, then invibetatail(a,b,p) = x

Domain a: 1e-10 to 1e+17 Domain b: 1e-10 to 1e+17

Domain p: 0 to 1 Range: 0 to 1

#### nbetaden(a,b,np,x)

Description: the probability density function of the noncentral beta distribution; 0 if x < 0 or x > 1

The probability density function of the noncentral beta distribution is defined as

$$\sum_{i=0}^{\infty} \frac{e^{-np/2} (np/2)^{j}}{\Gamma(j+1)} \left\{ \frac{\Gamma(a+b+j)}{\Gamma(a+j)\Gamma(b)} x^{a+j-1} (1-x)^{b-1} \right\}$$

where a and b are shape parameters, np is the noncentrality parameter, and x is the value of a beta random variable.

nbetaden(a,b,0,x) = betaden(a,b,x), but betaden() is the preferred function to use for the central beta distribution. nbetaden() is computed using an algorithm described in Johnson, Kotz, and Balakrishnan (1995).

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307

Domain np: 0 to 1,000

Domain x: -8e+307 to 8e+307; interesting domain is  $0 \le x \le 1$ 

Range: 0 to 8e+307

### nibeta(a,b,np,x)

Description: the cumulative noncentral beta distribution; 0 if x < 0; or 1 if x > 1

The cumulative noncentral beta distribution is defined as

$$I_x(a, b, np) = \sum_{j=0}^{\infty} \frac{e^{-np/2}(np/2)^j}{\Gamma(j+1)} I_x(a+j, b)$$

where a and b are shape parameters, np is the noncentrality parameter, x is the value of a beta random variable, and  $I_x(a,b)$  is the cumulative beta distribution, ibeta().

 $\mathtt{nibeta}(a,b,0,x) = \mathtt{ibeta}(a,b,x)$ , but  $\mathtt{ibeta}()$  is the preferred function to use for the central beta distribution.  $\mathtt{nibeta}()$  is computed using an algorithm described in Johnson, Kotz, and Balakrishnan (1995).

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307

Domain np: 0 to 10,000

Domain x: -8e+307 to 8e+307; interesting domain is  $0 \le x \le 1$ 

# invnibeta(a,b,np,p)

Description: the inverse cumulative noncentral beta distribution: if

nibeta(a,b,np,x) = p, then invibeta(a,b,np,p) = x

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307

Domain np: 0 to 1,000 Domain p: 0 to 1 Range: 0 to 1

## **Binomial distribution**

#### binomialp(n,k,p)

Description: the probability of observing floor(k) successes in floor(n) trials when the

probability of a success on one trial is p

Domain n: 1 to 1e+6 Domain k: 0 to n Domain p: 0 to 1 Range: 0 to 1

#### binomial $(n, k, \theta)$

Description: the probability of observing floor(k) or fewer successes in floor(n) trials when

the probability of a success on one trial is  $\theta$ ; 0 if k < 0; or 1 if k > n

Domain n: 0 to 1e+17

Domain k: -8e+307 to 8e+307; interesting domain is  $0 \le k < n$ 

Domain  $\theta$ : 0 to 1 Range: 0 to 1

#### binomialtail $(n, k, \theta)$

Description: the probability of observing floor(k) or more successes in floor(n) trials when

the probability of a success on one trial is  $\theta$ ; 1 if k < 0; or 0 if k > n

Domain n: 0 to 1e+17

Domain k: -8e+307 to 8e+307; interesting domain is  $0 \le k < n$ 

Domain  $\theta$ : 0 to 1 Range: 0 to 1

#### invbinomial(n, k, p)

Description: the inverse of the cumulative binomial; that is,  $\theta$  ( $\theta$  = probability of success on

one trial) such that the probability of observing floor(k) or fewer successes in

floor(n) trials is p

Domain n: 1 to 1e+17 Domain k: 0 to n-1

Domain p: 0 to 1 (exclusive)

#### invbinomialtail(n, k, p)

Description: the inverse of the right cumulative binomial; that is,  $\theta$  ( $\theta$  = probability of success

on one trial) such that the probability of observing floor(k) or more successes in

floor(n) trials is p

Domain n: 1 to 1e+17 Domain k: 1 to n

Domain p: 0 to 1 (exclusive)

Range: 0 to 1

# Cauchy distribution

#### cauchyden(a,b,x)

Description: the probability density of the Cauchy distribution with location parameter a and scale

parameter b

Domain *a*: -1e+300 to 1e+300 Domain *b*: 1e-100 to 1e+300 Domain *x*: -8e+307 to 8e+307 Range: 0 to 8e+307

#### cauchy (a,b,x)

Description: the cumulative Cauchy distribution with location parameter a and scale parameter b

Domain a: -1e+300 to 1e+300Domain b: 1e-100 to 1e+300Domain x: -8e+307 to 8e+307

Range: 0 to 1

#### cauchytail(a,b,x)

Description: the reverse cumulative (upper tail or survivor) Cauchy distribution with location

parameter a and scale parameter b

cauchytail(a,b,x) = 1 - cauchy(a,b,x)

Domain a: -1e+300 to 1e+300 Domain b: 1e-100 to 1e+300 Domain x: -8e+307 to 8e+307

Range: 0 to 1

# invcauchy(a,b,p)

Description: the inverse of cauchy(): if cauchy(a,b,x) = p, then

inveauchy (a,b,p) = xDomain a: -1e+300 to 1e+300Domain b: 1e-100 to 1e+300

Domain p: 0 to 1 (exclusive) Range: -8e+307 to 8e+307 invcauchytail(a,b,p)

Description: the inverse of cauchytail(): if cauchytail(a,b,x) = p, then

invcauchytail(a,b,p) = x

Domain *a*: -1e+300 to 1e+300 Domain *b*: 1e-100 to 1e+300 Domain *p*: 0 to 1 (exclusive) Range: -8e+307 to 8e+307

lncauchyden(a,b,x)

Description: the natural logarithm of the density of the Cauchy distribution with location parameter

a and scale parameter b

Domain a: -1e+300 to 1e+300 Domain b: 1e-100 to 1e+300 Domain x: -8e+307 to 8e+307 Range: -1650 to 230

Augustin-Louis Cauchy (1789–1857) was born in Paris, France. He obtained a degree in engineering with honors from École Polytechnique, where he would later teach mathematics. While working as a military engineer, he published two papers on polyhedra, one of which was a solution to a problem presented to him by Joseph-Louis Lagrange. In 1816, he won the Grand Prix for his work on wave propagation.

Cauchy's contributions were numerous and far reaching, as evident by the many concepts and theorems named after him. Some examples include the Cauchy criterion for convergence, Cauchy's theorem for finite groups, the Cauchy distribution, and the Cauchy stress tensor. His contributions were so vast that once all of his work was collected, it comprised 27 volumes. His name is engraved on the Eiffel Tower, along with 71 other scientists and mathematicians.

# $\chi^2$ and noncentral $\chi^2$ distributions

chi2den(df,x)

Description: the probability density of the  $\chi^2$  distribution with df degrees of freedom; 0 if x < 0

chi2den(df,x) = gammaden(df/2,2,0,x)

Domain df: 2e–10 to 2e+17 (may be nonintegral)

Domain x: -8e+307 to 8e+307

Range: 0 to 8e+307

chi2(df,x)

Description: the cumulative  $\chi^2$  distribution with df degrees of freedom; 0 if x < 0

chi2(df,x) = gammap(df/2,x/2)

Domain df: 2e-10 to 2e+17 (may be nonintegral)

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

#### chi2tail(df,x)

Description: the reverse cumulative (upper tail or survivor)  $\chi^2$  distribution with df degrees of

freedom; 1 if x < 0

chi2tail(df, x) = 1 - chi2(df, x) Domain df: 2e-10 to 2e+17 (may be nonintegral)

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

Range: 0 to 1

# invchi2(df,p)

Description: the inverse of chi2(): if chi2(df,x) = p, then invchi2(df,p) = x

Domain df: 2e–10 to 2e+17 (may be nonintegral)

Domain p: 0 to 1

Range: 0 to 8e+307

## invchi2tail(df,p)

Description: the inverse of chi2tail(): if chi2tail(df, x) = p, then invchi2tail(df, p) =

x

Domain df:  $\overset{\circ}{2}e-10$  to 2e+17 (may be nonintegral)

Domain p: 0 to 1

Range: 0 to 8e+307

#### nchi2den(df, np, x)

Description: the probability density of the noncentral  $\chi^2$  distribution; 0 if x < 0

df denotes the degrees of freedom, np is the noncentrality parameter, and x is the value of  $\chi^2$ .

nchi2den(df,0,x) = chi2den(df,x), but chi2den() is the preferred function to use for the central  $\chi^2$  distribution.

Domain df: 2e–10 to 1e+6 (may be nonintegral)

Domain np: 0 to 10,000

Domain x: -8e+307 to 8e+307

Range: 0 to 8e+307

#### nchi2(df, np, x)

Description: the cumulative noncentral  $\chi^2$  distribution; 0 if x < 0

The cumulative noncentral  $\chi^2$  distribution is defined as

$$\int_0^x \frac{e^{-t/2}\,e^{-np/2}}{2^{d\!f/2}}\,\sum_{j=0}^\infty \frac{t^{d\!f/2+j-1}\,np^j}{\Gamma(d\!f/2+j)\,2^{2j}\,j!}\,dt$$

where df denotes the degrees of freedom, np is the noncentrality parameter, and x is the value of  $\chi^2$ .

nchi2(df, 0, x) = chi2(df, x), but chi2() is the preferred function to use for the central  $\chi^2$  distribution

central  $\chi^2$  distribution. Domain df: 2e-10 to 1e+6 (may be nonintegral)

Domain np: 0 to 10,000

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

# nchi2tail(df, np, x)

Description: the reverse cumulative (upper tail or survivor) noncentral  $\chi^2$  distribution; 1 if x < 0

df denotes the degrees of freedom, np is the noncentrality parameter, and x is the

value of  $\chi^2$ .

Domain df: 2e–10 to 1e+6 (may be nonintegral)

Domain np: 0 to 10,000

Domain x: -8e+307 to 8e+307

Range: 0 to 1

# invnchi2(df, np, p)

Description: the inverse cumulative noncentral  $\chi^2$  distribution: if

nchi2(df, np, x) = p, then invnchi2(df, np, p) = x

Domain df: 2e–10 to 1e+6 (may be nonintegral)

Domain np: 0 to 10,000 Domain p: 0 to 1 Range: 0 to 8e+307

# invnchi2tail(df, np, p)

Description: the inverse reverse cumulative (upper tail or survivor) noncentral  $\chi^2$  distribution: if

nchi2tail(df, np, x) = p, then invnchi2tail(df, np, p) = x

Domain df: 2e–10 to 1e+6 (may be nonintegral)

Domain np: 0 to 10,000 Domain p: 0 to 1 Range: 0 to 8e+307

#### npnchi2(df,x,p)

Description: the noncentrality parameter, np, for noncentral  $\chi^2$ : if

nchi2(df, np, x) = p, then npnchi2(df, x, p) = np

Domain df: 2e–10 to 1e+6 (may be nonintegral)

Domain *x*: 0 to 8e+307 Domain *p*: 0 to 1 Range: 0 to 10,000

# Dunnett's multiple range distribution

# dunnettprob(k, df, x)

Description: the cumulative multiple range distribution that is used in Dunnett's multiple-comparison

method with k ranges and df degrees of freedom; 0 if x < 0

dunnettprob() is computed using an algorithm described in Miller (1981).

Domain k: 2 to 1e+6 Domain df: 2 to 1e+6

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

invdunnettprob(k, df, p)

Description: the inverse cumulative multiple range distribution that is used in Dunnett's multiple-

comparison method with k ranges and df degrees of freedom

If dunnettprob(k, df, x) = p, then involunnettprob(k, df, p) = x.

invdunnettprob() is computed using an algorithm described in Miller (1981).

Domain k: 2 to 1e+6 Domain df: 2 to 1e+6

Domain p: 0 to 1 (right exclusive)

Range: 0 to 8e+307

Charles William Dunnett (1921–2007) was a Canadian statistician best known for his work on multiple-comparison procedures. He was born in Windsor, Ontario, and graduated in mathematics and physics from McMaster University. After naval service in World War II, Dunnett's career included further graduate work, teaching, and research at Toronto, Columbia, the New York State Maritime College, the Department of National Health and Welfare in Ottawa, Cornell, Lederle Laboratories, and Aberdeen before he became Professor of Clinical Epidemiology and Biostatistics at McMaster University in 1974. He was President and Gold Medalist of the Statistical Society of Canada. Throughout his career, Dunnett took a keen interest in computing. According to Google Scholar, his 1955 paper on comparing treatments with a control has been cited over 4,000 times.

# **Exponential distribution**

exponentialden(b,x)

Description: the probability density function of the exponential distribution with scale b

The probability density function of the exponential distribution is

$$\frac{1}{b}\exp(-x/b)$$

where b is the scale and x is the value of an exponential variate.

Domain *b*: 1e–323 to 8e+307

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

Range: 1e-323 to 8e+307

exponential(b,x)

Description: the cumulative exponential distribution with scale b

The cumulative distribution function of the exponential distribution is

$$1 - \exp(-x/b)$$

for  $x \ge 0$  and 0 for x < 0, where b is the scale and x is the value of an exponential variate

The mean of the exponential distribution is b and its variance is  $b^2$ .

Domain *b*: 1e–323 to 8e+307

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

#### exponential tail(b,x)

Description: the reverse cumulative exponential distribution with scale b

The reverse cumulative distribution function of the exponential distribution is

$$\exp(-x/b)$$

where b is the scale and x is the value of an exponential variate.

Domain *b*: 1e–323 to 8e+307

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

Range: 0 to 1

# invexponential(b, p)

Description: the inverse cumulative exponential distribution with scale b: if

exponential (b,x) = p, then invexponential (b,p) = x

Domain *b*: 1e–323 to 8e+307

Domain p: 0 to 1

Range: 1e-323 to 8e+307

#### invexponential tail(b, p)

Description: the inverse reverse cumulative exponential distribution with scale b:

if exponentialtail(b,x) = p, then invexponentialtail(b,p) = x

Domain *b*:  $1e-32\bar{3}$  to 8e+307

Domain p: 0 to 1

Range: 1e-323 to 8e+307

#### F and noncentral F distributions

# Fden $(df_1, df_2, f)$

Description: the probability density function of the F distribution with  $df_1$  numerator and  $df_2$  denominator degrees of freedom; 0 if f < 0

The probability density function of the F distribution with  $df_1$  numerator and  $df_2$  denominator degrees of freedom is defined as

$$\mathtt{Fden}(d\!f_1,\!d\!f_2,\!f) = \frac{\Gamma(\frac{d\!f_1+d\!f_2}{2})}{\Gamma(\frac{d\!f_1}{2})\Gamma(\frac{d\!f_2}{2})} \bigg(\frac{d\!f_1}{d\!f_2}\bigg)^{\frac{d\!f_1}{2}} \cdot f^{\frac{d\!f_1}{2}-1} \bigg(1 + \frac{d\!f_1}{d\!f_2}f\bigg)^{-\frac{1}{2}(d\!f_1+d\!f_2)}$$

Domain  $df_1$ : 1e–323 to 8e+307 (may be nonintegral) Domain  $df_2$ : 1e–323 to 8e+307 (may be nonintegral)

Domain f: -8e+307 to 8e+307; interesting domain is  $f \ge 0$ 

Range: 0 to 8e+307

 $F(df_1, df_2, f)$ 

Description: the cumulative F distribution with  $df_1$  numerator and  $df_2$  denominator degrees of

freedom: F( $df_1$ , $df_2$ ,f) =  $\int_0^f$  Fden( $df_1$ , $df_2$ ,t) dt; 0 if f<0 Domain  $df_1$ : 2e-10 to 2e+17 (may be nonintegral)

Domain  $df_1$ : 2e–10 to 2e+17 (may be nonintegral) Domain  $df_2$ : 2e–10 to 2e+17 (may be nonintegral)

Domain f: -8e+307 to 8e+307; interesting domain is  $f \ge 0$ 

Range: 0 to 1

Ftail( $df_1, df_2, f$ )

Description: the reverse cumulative (upper tail or survivor) F distribution with  $df_1$  numerator and  $df_2$  denominator degrees of freedom; 1 if f < 0

 $Ftail(df_1, df_2, f) = 1 - F(df_1, df_2, f).$ 

Domain  $df_1$ : 2e–10 to 2e+17 (may be nonintegral) Domain  $df_2$ : 2e–10 to 2e+17 (may be nonintegral)

Domain f: -8e+307 to 8e+307; interesting domain is  $f \ge 0$ 

Range: 0 to 1

 $invF(df_1, df_2, p)$ 

Description: the inverse cumulative F distribution: if  $F(df_1, df_2, f) = p$ , then

 $invF(df_1, df_2, p) = f$ 

Domain  $df_1$ : 2e-10 to 2e+17 (may be nonintegral) Domain  $df_2$ : 2e-10 to 2e+17 (may be nonintegral)

Domain p: 0 to 1

Range: 0 to 8e+307

 $invFtail(df_1, df_2, p)$ 

Description: the inverse reverse cumulative (upper tail or survivor) F distribution:

if Ftail( $df_1, df_2, f$ ) = p, then invFtail( $df_1, df_2, p$ ) = f

Domain  $df_1$ : 2e-10 to 2e+17 (may be nonintegral) Domain  $df_2$ : 2e-10 to 2e+17 (may be nonintegral)

Domain p: 0 to 1

Range: 0 to 8e+307

 $nFden(df_1, df_2, np, f)$ 

Description: the probability density function of the noncentral F distribution with  $df_1$  numerator and  $df_2$  denominator degrees of freedom and noncentrality parameter np; 0 if f < 0

nFden $(df_1, df_2, 0, f)$  = Fden $(df_1, df_2, f)$ , but Fden() is the preferred function to use for the central F distribution.

Also, if F follows the noncentral F distribution with  $df_1$  and  $df_2$  degrees of freedom and noncentrality parameter np, then

$$\frac{df_1F}{df_2 + df_1F}$$

follows a noncentral beta distribution with shape parameters  $a=df_1/2$ ,  $b=df_2/2$ , and noncentrality parameter np, as given in nbetaden(). nFden() is computed based on this relationship.

Domain  $df_1$ : 1e–323 to 8e+307 (may be nonintegral) Domain  $df_2$ : 1e–323 to 8e+307 (may be nonintegral)

Domain np: 0 to 1,000

Domain f: -8e+307 to 8e+307; interesting domain is  $f \ge 0$ 

Range: 0 to 8e+307

 $nF(df_1, df_2, np, f)$ 

Description: the cumulative noncentral F distribution with  $df_1$  numerator and  $df_2$  denominator degrees of freedom and noncentrality parameter np; 0 if f < 0

$$\mathtt{nF}(\mathit{df}_1,\mathit{df}_2,\mathtt{0},f) = \mathtt{F}(\mathit{df}_1,\mathit{df}_2,f)$$

nF() is computed using nibeta() based on the relationship between the noncentral beta and noncentral F distributions: nF( $df_1$ ,  $df_2$ , np, f) = nibeta( $df_1/2$ ,  $df_2/2$ , np,  $df_1 \times f/\{(df_1 \times f) + df_2\}$ ).

Domain  $df_1$ : 2e–10 to 1e+8 Domain  $df_2$ : 2e–10 to 1e+8 Domain np: 0 to 10,000

Domain f: -8e+307 to 8e+307

Range: 0 to 1

 $nFtail(df_1, df_2, np, f)$ 

Description: the reverse cumulative (upper tail or survivor) noncentral F distribution with  $df_1$  numerator and  $df_2$  denominator degrees of freedom and noncentrality parameter np; 1 if f < 0

nFtail() is computed using nibeta() based on the relationship between the noncentral beta and F distributions. See Johnson, Kotz, and Balakrishnan (1995) for more details

more details. Domain  $df_1$ : 1e-323 to 8e+307 (may be nonintegral) Domain  $df_2$ : 1e-323 to 8e+307 (may be nonintegral)

Domain np: 0 to 1,000

Domain f: -8e+307 to 8e+307; interesting domain is  $f \ge 0$ 

 $invnF(df_1, df_2, np, p)$ 

Description: the inverse cumulative noncentral F distribution: if

 $\operatorname{nF}(df_1, df_2, np, f) = p$ , then  $\operatorname{invnF}(df_1, df_2, np, p) = f$ 

Domain  $df_1$ : 1e–6 to 1e+6 (may be nonintegral) Domain  $df_2$ : 1e–6 to 1e+6 (may be nonintegral)

Domain np: 0 to 10,000 Domain p: 0 to 1 Range: 0 to 8e+307

 $invnFtail(df_1, df_2, np, p)$ 

Description: the inverse reverse cumulative (upper tail or survivor) noncentral F distribution: if

 $\mathsf{nFtail}(df_1, df_2, np, f) = p$ , then  $\mathsf{invnFtail}(df_1, df_2, np, p) = f$ 

Domain  $df_1$ : 1e–323 to 8e+307 (may be nonintegral) Domain  $df_2$ : 1e–323 to 8e+307 (may be nonintegral)

Domain np: 0 to 1,000 Domain p: 0 to 1 Range: 0 to 8e+307

 $\mathtt{npnF}(df_1, df_2, f, p)$ 

Description: the noncentrality parameter, np, for the noncentral F: if

 $\operatorname{nF}(df_1, df_2, np, f) = p$ , then  $\operatorname{npnF}(df_1, df_2, f, p) = np$ 

Domain  $df_1$ : 2e-10 to 1e+6 (may be nonintegral) Domain  $df_2$ : 2e-10 to 1e+6 (may be nonintegral)

Domain f: 0 to 8e+307 Domain p: 0 to 1 Range: 0 to 1,000

#### Gamma distribution

gammaden(a,b,g,x)

Description: the probability density function of the gamma distribution; 0 if x < g

The probability density function of the gamma distribution is defined by

$$\frac{1}{\Gamma(a)b^{a}}(x-g)^{a-1}e^{-(x-g)/b}$$

where a is the shape parameter, b is the scale parameter, and g is the location

parameter.

Domain a: 1e-323 to 8e+307 Domain b: 1e-323 to 8e+307 Domain g: -8e+307 to 8e+307

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge q$ 

Range: 0 to 8e+307

#### gammap(a,x)

Description: the cumulative gamma distribution with shape parameter a; 0 if x < 0

The cumulative gamma distribution with shape parameter a is defined by

$$\frac{1}{\Gamma(a)} \int_0^x e^{-t} t^{a-1} dt$$

The cumulative Poisson (the probability of observing k or fewer events if the expected is x) can be evaluated as 1-gammap(k+1,x). The reverse cumulative (the probability of observing k or more events) can be evaluated as gammap(k,x). See Press et al. (2007, 259–266) for a more complete description and for suggested uses for this function.

gammap() is also known as the incomplete gamma function (ratio).

Probabilities for the three-parameter gamma distribution (see gammaden()) can be calculated by shifting and scaling x; that is, gammap(a, (x-q)/b).

Domain a: 1e–10 to 1e+17

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

Range: 0 to 1

### gammaptail(a,x)

Description: the reverse cumulative (upper tail or survivor) gamma distribution with shape parameter a; 1 if x < 0

The reverse cumulative (upper tail or survivor) gamma distribution with shape parameter a is defined by

$$\mathtt{gammaptail}(a,x) = 1 - \mathtt{gammap}(a,x) = \int_x^\infty \mathtt{gammaden}(a,t) \, dt$$

gammaptail() is also known as the complement to the incomplete gamma function (ratio).

Domain a: 1e-10 to 1e+17

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

Range: 0 to 1

### invgammap(a,p)

Description: the inverse cumulative gamma distribution: if gammap (a,x) = p,

then invgammap(a,p) = x

Domain a: 1e–10 to 1e+17

Domain p: 0 to 1

Range: 0 to 8e+307

## invgammaptail(a,p)

Description: the inverse reverse cumulative (upper tail or survivor) gamma distribution: if

gammaptail(a,x) = p, then invgammaptail(a,p) = x

Domain a: 1e–10 to 1e+17

Domain p: 0 to 1

Range: 0 to 8e+307

#### dgammapda(a,x)

Description:  $\frac{\partial P(a,x)}{\partial a}$ , where  $P(a,x)=\mathrm{gammap}(a,x)$ ; 0 if x<0

Domain a: 1e-7 to 1e+17

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

Range: -16 to 0

## dgammapdada(a,x)

Description:  $\frac{\partial^2 P(a,x)}{\partial a^2}$ , where  $P(a,x)=\mathrm{gammap}(a,x)$ ; 0 if x<0

Domain a: 1e-7 to 1e+17

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

Range: -0.02 to 4.77e+5

## dgammapdadx(a,x)

Description:  $\frac{\partial^2 P(a,x)}{\partial a \partial x}$ , where P(a,x) = gammap(a,x); 0 if x < 0

Domain a: 1e-7 to 1e+17

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

Range: -0.04 to 8e+307

#### dgammapdx(a,x)

Description:  $\frac{\partial P(a,x)}{\partial x}$ , where P(a,x) = gammap(a,x); 0 if x < 0

Domain a: 1e–10 to 1e+17

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

Range: 0 to 8e+307

# dgammapdxdx(a,x)

Description:  $\frac{\partial^2 P(a,x)}{\partial x^2}$ , where P(a,x) = gammap(a,x); 0 if x < 0

Domain a: 1e–10 to 1e+17

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge 0$ 

Range: 0 to 1e+40

## lnigammaden(a,b,x)

Description: the natural logarithm of the inverse gamma density, where a is the shape parameter

and b is the scale parameter

Domain a: 1e–300 to 1e+300 Domain b: 1e–300 to 1e+300 Domain x: 1e–300 to 8e+307

Range: -8e+307 to 8e+307

# Hypergeometric distribution

hypergeometricp(N, K, n, k)

Description: the hypergeometric probability of k successes out of a sample of size n, from a

population of size N containing K elements that have the attribute of interest

Success is obtaining an element with the attribute of interest.

Domain N: 2 to 1e+5 Domain K: 1 to N-1Domain n: 1 to N-1

Domain k:  $\max(0, n - N + K)$  to  $\min(K, n)$ 

Range: 0 to 1 (right exclusive)

hypergeometric (N, K, n, k)

Description: the cumulative probability of the hypergeometric distribution

N is the population size, K is the number of elements in the population that have the attribute of interest, and n is the sample size. Returned is the probability of observing k or fewer elements from a sample of size n that have the attribute of interest.

Domain N: 2 to 1e+5 Domain K: 1 to N-1Domain n: 1 to N-1

Domain k:  $\max(0, n - N + K)$  to  $\min(K, n)$ 

Range: 0 to 1

#### Inverse Gaussian distribution

igaussianden(m,a,x)

Description: the probability density of the inverse Gaussian distribution with mean m and shape

parameter a; 0 if  $x \le 0$ 

Domain m: 1e-323 to 8e+307 Domain a: 1e-323 to 8e+307 Domain x: -8e+307 to 8e+307

Range: 0 to 8e+307

igaussian(m,a,x)

Description: the cumulative inverse Gaussian distribution with mean m and shape parameter a; 0

if  $x \leq 0$ 

Domain m:  $1e-3\overline{23}$  to 8e+307Domain a: 1e-323 to 8e+307Domain x: -8e+307 to 8e+307

Range: 0 to 1

igaussiantail(m,a,x)

Description: the reverse cumulative (upper tail or survivor) inverse Gaussian distribution with

mean m and shape parameter a; 1 if  $x \le 0$ 

igaussiantail(m,a,x) = 1 - igaussian(m,a,x)

Domain m: 1e–323 to 8e+307 Domain a: 1e–323 to 8e+307 Domain x: -8e+307 to 8e+307

#### invigaussian(m,a,p)

Description: the inverse of igaussian(): if

igaussian(m,a,x)=p, then invigaussian(m,a,p)=x

Domain *m*: 1e–323 to 8e+307 Domain *a*: 1e–323 to 1e+8 Domain *p*: 0 to 1 (exclusive) Range: 0 to 8e+307

### invigaussiantail(m, a, p)

Description: the inverse of igaussiantail(): if

igaussiantail(m,a,x) = p, then invigaussiantail(m,a,p) = x

Domain *m*: 1e–323 to 8e+307 Domain *a*: 1e–323 to 1e+8 Domain *p*: 0 to 1 (exclusive) Range: 0 to 8e+307

### lnigaussianden(m,a,x)

Description: the natural logarithm of the inverse Gaussian density with mean m and shape

parameter a

Domain *m*: 1e–323 to 8e+307 Domain *a*: 1e–323 to 8e+307 Domain *x*: 1e–323 to 8e+307 Range: –8e+307 to 8e+307

# Laplace distribution

# laplaceden(m,b,x)

Description: the probability density of the Laplace distribution with mean m and scale parameter b

Domain *m*: -8e+307 to 8e+307 Domain *b*: 1e-307 to 8e+307 Domain *x*: -8e+307 to 8e+307 Range: 0 to 8e+307

# laplace(m,b,x)

Description: the cumulative Laplace distribution with mean m and scale parameter b

Domain m: -8e+307 to 8e+307 Domain b: 1e-307 to 8e+307 Domain x: -8e+307 to 8e+307

Range: 0 to 1

# laplacetail(m,b,x)

Description: the reverse cumulative (upper tail or survivor) Laplace distribution with mean m and

scale parameter b

laplacetail(m,b,x) = 1 - laplace(m,b,x)

Domain m: -8e+307 to 8e+307Domain b: 1e-307 to 8e+307Domain x: -8e+307 to 8e+307

# invlaplace(m,b,p)

Description: the inverse of laplace(): if laplace(m, b, x) = p, then

invlaplace(m,b,p) = x

Domain m: -8e+307 to 8e+307Domain b: 1e-307 to 8e+307 Domain p: 0 to 1 (exclusive) Range: -8e+307 to 8e+307

### invlaplacetail(m,b,p)

Description: the inverse of laplacetail(): if laplacetail(m, b, x) = p,

then invlaplacetail(m, b, p) = x

Domain m: -8e+307 to 8e+307Domain *b*: 1e-307 to 8e+307 Domain p: 0 to 1 (exclusive) -8e+307 to 8e+307Range:

## lnlaplaceden(m,b,x)

Description: the natural logarithm of the density of the Laplace distribution with mean m and

scale parameter b

Domain m: -8e+307 to 8e+307Domain b: 1e-307 to 8e+307 Domain x: -8e+307 to 8e+307Range: -8e+307 to 707

# Logistic distribution

#### logisticden(x)

Description: the density of the logistic distribution with mean 0 and standard deviation  $\pi/\sqrt{3}$ 

logisticden(x) = logisticden(1,x) = logisticden(0,1,x), where x is

the value of a logistic random variable.

Domain x: -8e+307 to 8e+307

Range: 0 to 0.25

#### logisticden(s,x)

Description: the density of the logistic distribution with mean 0, scale s, and standard deviation

 $s\pi/\sqrt{3}$ 

logisticden(s,x) = logisticden(0,s,x), where s is the scale and x is the

value of a logistic random variable.

1e-323 to 8e+307 Domain s: -8e+307 to 8e+307Domain x:

Range: 0 to 8e + 307

### logisticden(m,s,x)

Description: the density of the logistic distribution with mean m, scale s, and standard deviation

 $s\pi/\sqrt{3}$ 

The density of the logistic distribution is defined as

$$\frac{\exp\{-(x-m)/s\}}{s[1+\exp\{-(x-m)/s\}]^2}$$

where m is the mean, s is the scale, and x is the value of a logistic random variable.

Domain m: -8e+307 to 8e+307Domain s: 1e-323 to 8e+307Domain x: -8e+307 to 8e+307Range: 0 to 8e+307

#### logistic(x)

Description: the cumulative logistic distribution with mean 0 and standard deviation  $\pi/\sqrt{3}$ 

logistic(x) = logistic(1,x) = logistic(0,1,x), where x is the value of a logistic random variable.

Domain x: -8e+307 to 8e+307

Range: 0 to 1

#### logistic(s,x)

Description: the cumulative logistic distribution with mean 0, scale s, and standard deviation  $s\pi/\sqrt{3}$ 

logistic(s, x) = logistic(0, s, x), where s is the scale and x is the value of a logistic random variable.

Domain s: 1e–323 to 8e+307 Domain x: -8e+307 to 8e+307

Range: 0 to 1

#### logistic(m,s,x)

Description: the cumulative logistic distribution with mean m, scale s, and standard deviation  $s\pi/\sqrt{3}$ 

The cumulative logistic distribution is defined as

$$[1 + \exp{-(x-m)/s}]^{-1}$$

where m is the mean, s is the scale, and x is the value of a logistic random variable.

Domain m: -8e+307 to 8e+307Domain s: 1e-323 to 8e+307Domain x: -8e+307 to 8e+307

#### logistictail(x)

Description: the reverse cumulative logistic distribution with mean 0 and standard deviation  $\pi/\sqrt{3}$ 

logistictail(x) = logistictail(1,x) = logistictail(0,1,x), where x is the value of a logistic random variable.

Domain x: -8e+307 to 8e+307

Range: 0 to 1

# logistictail(s,x)

Description: the reverse cumulative logistic distribution with mean 0, scale s, and standard deviation

$$s\pi/\sqrt{3}$$

logistictail(s,x) = logistictail(0,s,x), where s is the scale and x is the value of a logistic random variable.

Domain s: 1e-323 to 8e+307Domain x: -8e+307 to 8e+307

Range: 0 to 1

#### logistictail(m,s,x)

Description: the reverse cumulative logistic distribution with mean m, scale s, and standard deviation  $s\pi/\sqrt{3}$ 

The reverse cumulative logistic distribution is defined as

$$[1 + \exp\{(x - m)/s\}]^{-1}$$

where m is the mean, s is the scale, and x is the value of a logistic random variable.

Domain m: -8e+307 to 8e+307Domain s: 1e-323 to 8e+307Domain x: -8e+307 to 8e+307

Range: 0 to 1

# invlogistic(p)

Description: the inverse cumulative logistic distribution: if logistic(x) = p,

then invlogistic(p) = x

Domain p: 0 to 1

Range: -8e+307 to 8e+307

# invlogistic(s,p)

Description: the inverse cumulative logistic distribution: if logistic(s,x) = p, then

invlogistic(s,p) = x

Domain s: 1e–323 to 8e+307

Domain p: 0 to 1

Range: -8e+307 to 8e+307

```
invlogistic(m,s,p)
  Description: the inverse cumulative logistic distribution: if logistic(m,s,x) = p, then
              invlogistic(m,s,p) = x
  Domain m: -8e+307 to 8e+307
  Domain s:
             1e-323 to 8e+307
  Domain p: 0 to 1
              -8e+307 to 8e+307
  Range:
invlogistictail(p)
  Description: the inverse reverse cumulative logistic distribution: if
              logistictail(x) = p, then invlogistictail(p) = x
  Domain p:
              0 to 1
              -8e+307 to 8e+307
  Range:
invlogistictail(s,p)
  Description: the inverse reverse cumulative logistic distribution: if
              logistictail(s,x) = p, then invlogistictail(s,p) = x
  Domain s:
             1e-323 to 8e+307
  Domain p:
             0 to 1
  Range:
              -8e+307 to 8e+307
invlogistictail(m,s,p)
  Description: the inverse reverse cumulative logistic distribution: if
              logistictail(m,s,x) = p, then
              invlogistictail(m,s,p) = x
  Domain m: -8e+307 to 8e+307
  Domain s: 1e-323 to 8e+307
  Domain p: 0 to 1
  Range:
              -8e+307 to 8e+307
```

# **Negative binomial distribution**

```
nbinomialp(n,k,p)
```

Description: the negative binomial probability

When n is an integer, nbinomialp() returns the probability of observing exactly floor(k) failures before the nth success when the probability of a success on one trial is p.

Domain n: 1e–10 to 1e+6 (can be nonintegral)

Domain k: 0 to 1e+10

Domain p: 0 to 1 (left exclusive)

#### nbinomial(n,k,p)

Description: the cumulative probability of the negative binomial distribution

n can be nonintegral. When n is an integer, nbinomial() returns the probability of observing k or fewer failures before the nth success, when the probability of a success on one trial is p.

The negative binomial distribution function is evaluated using ibeta().

Domain n: 1e–10 to 1e+17 (can be nonintegral)

Domain k: 0 to  $2^{53} - 1$ 

Domain p: 0 to 1 (left exclusive)

Range: 0 to 1

### nbinomialtail(n, k, p)

Description: the reverse cumulative probability of the negative binomial distribution

When n is an integer, nbinomialtail() returns the probability of observing k or more failures before the nth success, when the probability of a success on one trial is p.

The reverse negative binomial distribution function is evaluated using ibetatail().

Domain n: 1e–10 to 1e+17 (can be nonintegral)

Domain k: 0 to  $2^{53} - 1$ 

Domain p: 0 to 1 (left exclusive)

Range: 0 to 1

#### invnbinomial (n, k, q)

Description: the value of the negative binomial parameter, p, such that  $q = \mathtt{nbinomial}(n, k, p)$ 

invnbinomial() is evaluated using invibeta().

Domain n: 1e-10 to 1e+17 (can be nonintegral)

Domain k: 0 to  $2^{53} - 1$ Domain q: 0 to 1 (exclusive)

Range: 0 to 1

### invnbinomialtail(n, k, q)

Description: the value of the negative binomial parameter, p, such that

q = nbinomialtail(n, k, p)

invnbinomialtail() is evaluated using invibetatail().

Domain n: 1e–10 to 1e+17 (can be nonintegral)

Domain k: 1 to  $2^{53} - 1$ Domain q: 0 to 1 (exclusive) Range: 0 to 1 (exclusive)

# Normal (Gaussian), binormal, and multivariate normal distributions

#### normalden(z)

Description: the standard normal density, N(0,1)

Domain: -8e+307 to 8e+307 Range: 0 to 0.39894 ...

#### $normalden(x, \sigma)$

Description: the normal density with mean 0 and standard deviation  $\sigma$ 

normalden(x,1) = normalden(x) and  $normalden(x, \sigma) = normalden(x/\sigma)/\sigma$ .

Domain x: -8e+307 to 8e+307Domain  $\sigma$ : 1e–308 to 8e+307 0 to 8e+307 Range:

### $normalden(x, \mu, \sigma)$

Description: the normal density with mean  $\mu$  and standard deviation  $\sigma$ ,  $N(\mu, \sigma^2)$ 

normalden(x,0,s) = normalden(x,s) and normalden $(x, \mu, \sigma)$  = normalden $((x - \mu)/\sigma)/\sigma$ . In general,

$$\texttt{normalden}(z,\!\mu,\!\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left\{\frac{(z-\mu)}{\sigma}\right\}^2}$$

Domain x: -8e+307 to 8e+307Domain  $\mu$ : -8e+307 to 8e+307 Domain  $\sigma$ : 1e–308 to 8e+307 Range: 0 to 8e+307

#### normal(z)

Description: the cumulative standard normal distribution

normal(z) =  $\int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$  -8e+307 to 8e+307

Domain:

Range: 0 to 1

## invnormal(p)

Description: the inverse cumulative standard normal distribution: if normal(z) = p, then

invnormal(p) = z

1e-323 to  $1 - 2^{-53}$ Domain:

-38.449394 to 8.2095362 Range:

#### lnnormalden(z)

Description: the natural logarithm of the standard normal density, N(0,1)

Domain: -1e+154 to 1e+154

-5e+307 to -0.91893853 = lnnormalden(0)Range:

# lnnormalden $(x,\sigma)$

Description: the natural logarithm of the normal density with mean 0 and standard deviation  $\sigma$ 

lnnormalden(x, 1) = lnnormalden(x) and

 $lnnormalden(x,\sigma) = lnnormalden(x/\sigma) - ln(\sigma).$ 

Domain x: -8e+307 to 8e+307Domain  $\sigma$ : 1e–323 to 8e+307 -5e+307 to 742.82799Range:

### lnnormalden( $x, \mu, \sigma$ )

Description: the natural logarithm of the normal density with mean  $\mu$  and standard deviation  $\sigma$ ,  $N(\mu, \sigma^2)$ 

Innormalden(x,0,s) = Innormalden(x,s) and Innormalden(x, $\mu$ , $\sigma$ ) = Innormalden( $(x - \mu)/\sigma$ ) -  $\ln(\sigma)$ . In general,

$${\tt lnnormalden}(z,\mu,\sigma) = \ln\left[\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left\{\frac{(z-\mu)}{\sigma}\right\}^2}\right]$$

Domain x: -8e+307 to 8e+307Domain  $\mu$ : -8e+307 to 8e+307Domain  $\sigma$ : 1e-323 to 8e+307Range: 1e-323 to 8e+307

#### lnnormal(z)

Description: the natural logarithm of the cumulative standard normal distribution

$$\operatorname{lnnormal}(z) = \ln \left( \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \right)$$

Domain: -1e+99 to 8e+307Range: -5e+197 to 0

# binormal( $h, k, \rho$ )

Description: the joint cumulative distribution  $\Phi(h,k,\rho)$  of bivariate normal with correlation  $\rho$ 

Cumulative over  $(-\infty, h] \times (-\infty, k]$ :

$$\Phi(h,k,\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{h} \int_{-\infty}^{k} \exp\left\{-\frac{1}{2(1-\rho^2)} \left(x_1^2 - 2\rho x_1 x_2 + x_2^2\right)\right\} dx_1 dx_2$$

Domain h: -8e+307 to 8e+307Domain k: -8e+307 to 8e+307

Domain  $\rho$ : -1 to 1 Range: 0 to 1

# ${\tt lnmvnormalden}(M, V, X)$

Description: the natural logarithm of the multivariate normal density

M is the mean vector, V is the covariance matrix, and X is the random vector.

Domain M:  $1 \times n$  and  $n \times 1$  vectors

Domain  $V: n \times n$ , positive-definite, symmetric matrices

Domain X:  $1 \times n$  and  $n \times 1$  vectors Range: -8e+307 to 8e+307

#### Poisson distribution

```
poissonp(m,k)
```

Description: the probability of observing floor(k) outcomes that are distributed as Poisson with

mean m

The Poisson probability function is evaluated using gammaden().

Domain m: 1e-10 to 1e+8 Domain k: 0 to 1e+9 Range: 0 to 1

#### poisson(m,k)

Description: the probability of observing floor(k) or fewer outcomes that are distributed as

Poisson with mean m

The Poisson distribution function is evaluated using gammaptail().

Domain m: 1e–10 to  $2^{53} - 1$ 0 to  $2^{53} - 1$ Domain k: Range: 0 to 1

#### poissontail(m,k)

Description: the probability of observing floor(k) or more outcomes that are distributed as

Poisson with mean m

The reverse cumulative Poisson distribution function is evaluated using gammap().

Domain m: 1e–10 to  $2^{53} - 1$ Domain k: 0 to  $2^{53} - 1$ Range: 0 to 1

### invpoisson(k,p)

Description: the Poisson mean such that the cumulative Poisson distribution evaluated at k is p:

if poisson(m,k) = p, then invpoisson(k,p) = m

The inverse Poisson distribution function is evaluated using invgammaptail().

0 to  $2^{53}-1$ Domain k: 0 to 1 (exclusive) Domain p: 1.110e-16 to 2<sup>53</sup> Range:

### invpoissontail(k,q)

Description: the Poisson mean such that the reverse cumulative Poisson distribution evaluated at

k is q: if poissontail(m,k) = q, then invpoissontail(k,q) = m

The inverse of the reverse cumulative Poisson distribution function is evaluated using invgammap().

0 to  $2^{53} - 1$ Domain k: 0 to 1 (exclusive) Domain q: 0 to  $2^{53}$  (left exclusive) Range:

## Student's t and noncentral Student's t distributions

tden(df,t)

Description: the probability density function of Student's t distribution

$$\mathtt{tden}(\mathit{df}\,,t) = \frac{\Gamma\{(\mathit{df}+1)/2\}}{\sqrt{\pi \mathit{df}}\Gamma(\mathit{df}/2)} \cdot \left(1 + t^2/\mathit{df}\right)^{-(\mathit{df}+1)/2}$$

Domain df: 1e-323 to 8e+307 (may be nonintegral)

Domain t: -8e+307 to 8e+307 Range: 0 to 0.39894 ...

t(df,t)

Description: the cumulative Student's t distribution with df degrees of freedom

Domain df: 2e–10 to 2e+17 (may be nonintegral)

Domain t; -8e+307 to 8e+307

Range: 0 to 1

ttail(df,t)

Description: the reverse cumulative (upper tail or survivor) Student's t distribution; the probability T > t

$$\mathtt{ttail}(df,t) = \int_{t}^{\infty} \frac{\Gamma\{(df+1)/2\}}{\sqrt{\pi df} \Gamma(df/2)} \cdot (1 + x^2/df)^{-(df+1)/2} \ dx$$

Domain df: 2e-10 to 2e+17 (may be nonintegral)

Domain t: -8e+307 to 8e+307

Range: 0 to 1

 $\mathtt{invt}(df,p)$ 

Description: the inverse cumulative Student's t distribution: if t(df,t) = p, then invt(df,p) = t

Domain df: 2e-10 to 2e+17 (may be nonintegral)

Domain p: 0 to 1

Range: -8e+307 to 8e+307

invttail(df,p)

Description: the inverse reverse cumulative (upper tail or survivor) Student's t distribution: if

ttail(df,t) = p, then invttail(df,p) = t

Domain df: 2e-10 to 2e+17 (may be nonintegral)

Domain p: 0 to 1

Range: -8e+307 to 8e+307

invnt(df, np, p)

Description: the inverse cumulative noncentral Student's t distribution: if nt(df, np, t) = p, then

invnt(df, np, p) = t

Domain df: 1 to 1e+6 (may be nonintegral)

Domain np: -1,000 to 1,000 Domain p: 0 to 1

Range: -8e+307 to 8e+307

invnttail(df, np, p)

Description: the inverse reverse cumulative (upper tail or survivor) noncentral Student's

t distribution: if nttail(df, np, t) = p, then invnttail(df, np, p) = t

Domain df: 1 to 1e+6 (may be nonintegral)

Domain np: -1,000 to 1,000

Domain p: 0 to 1

Range: -8e+10 to 8e+10

ntden(df, np, t)

Description: the probability density function of the noncentral Student's

t distribution with df degrees of freedom and noncentrality parameter np

Domain df: 1e–100 to 1e+10 (may be nonintegral)

Domain np: -1,000 to 1,000Domain t: -8e+307 to 8e+307Range: 0 to 0.39894...

nt(df, np, t)

Description: the cumulative noncentral Student's t distribution with df degrees of freedom and noncentrality parameter np

nt(df,0,t) = t(df,t).

Domain df: 1e–100 to 1e+10 (may be nonintegral)

Domain np: -1,000 to 1,000Domain t: -8e+307 to 8e+307

Range: 0 to 1

 $\mathtt{nttail}(\mathit{df}, np, t)$ 

Description: the reverse cumulative (upper tail or survivor) noncentral Student's t distribution with

 $d\!f$  degrees of freedom and noncentrality parameter np

Domain df: 1e-100 to 1e+10 (may be nonintegral)

Domain np: -1,000 to 1,000Domain t: -8e+307 to 8e+307

Range: 0 to 1

npnt(df,t,p)

Description: the noncentrality parameter, np, for the noncentral Student's

t distribution: if nt(df, np, t) = p, then npnt(df, t, p) = np

Domain df: 1e–100 to 1e+8 (may be nonintegral)

Domain t: -8e+307 to 8e+307

Domain p: 0 to 1

Range: -1,000 to 1,000

# Tukey's Studentized range distribution

tukeyprob(k, df, x)

Description: the cumulative Tukey's Studentized range distribution with k ranges and df degrees

of freedom; 0 if x < 0

If df is a missing value, then the normal distribution is used instead of Student's t.

tukeyprob() is computed using an algorithm described in Miller (1981).

Domain k: 2 to 1e+6 Domain df: 2 to 1e+6

Domain x: -8e+307 to 8e+307

Range: 0 to 1

invtukeyprob(k, df, p)

Description: the inverse cumulative Tukey's Studentized range distribution with k ranges and df

degrees of freedom

If df is a missing value, then the normal distribution is used instead of Student's t.

If tukeyprob(k, df, x) = p, then invtukeyprob(k, df, p) = x.

invtukeyprob() is computed using an algorithm described in Miller (1981).

Domain k: 2 to 1e+6 Domain df: 2 to 1e+6 Domain p: 0 to 1 Range: 0 to 8e+307

# Weibull distribution

weibullden(a,b,x)

Description: the probability density function of the Weibull distribution with shape a and scale b

weibullden(a, b, x) = weibullden(a, b, 0, x), where a is the shape, b is the

scale, and x is the value of Weibull random variable.

Domain a: 1e–323 to 8e+307 Domain b: 1e–323 to 8e+307 Domain x: 1e–323 to 8e+307 Range: 0 to 8e+307

# weibullden(a,b,g,x)

Description: the probability density function of the Weibull distribution with shape a, scale b, and location q

The probability density function of the generalized Weibull distribution is defined as

$$\frac{a}{b} \left( \frac{x-g}{b} \right)^{a-1} \exp \left\{ -\left( \frac{x-g}{b} \right)^a \right\}$$

for  $x \ge g$  and 0 for x < g, where a is the shape, b is the scale, g is the location parameter, and x is the value of a generalized Weibull random variable.

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307 Domain *g*: –8e+307 to 8e+307

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge g$ 

Range: 0 to 8e+307

#### weibull(a,b,x)

Description: the cumulative Weibull distribution with shape a and scale b

weibull(a, b, x) = weibull(a, b, 0, x), where a is the shape, b is the scale, and x is the value of Weibull random variable.

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307 Domain *x*: 1e–323 to 8e+307

Range: 0 to 1

#### weibull(a,b,g,x)

Description: the cumulative Weibull distribution with shape a, scale b, and location g

The cumulative Weibull distribution is defined as

$$1 - \exp\left[-\left(\frac{x-g}{b}\right)^a\right]$$

for  $x \ge g$  and 0 for x < g, where a is the shape, b is the scale, g is the location parameter, and x is the value of a Weibull random variable.

The mean of the Weibull distribution is  $g + b\Gamma\{(a+1)/a\}$  and its variance is  $b^2\left(\Gamma\{(a+2)/a\} - [\Gamma\{(a+1)/a\}]^2\right)$  where  $\Gamma()$  is the gamma function described in lngamma().

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307 Domain *q*: –8e+307 to 8e+307

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge g$ 

#### weibulltail(a,b,x)

Description: the reverse cumulative Weibull distribution with shape a and scale b

weibulltail(a,b,x) = weibulltail(a,b,0,x), where a is the shape, b is the scale, and x is the value of a Weibull random variable.

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307 Domain *x*: 1e–323 to 8e+307

Range: 0 to 1

# weibulltail(a,b,g,x)

Description: the reverse cumulative Weibull distribution with shape a, scale b, and location g

The reverse cumulative Weibull distribution is defined as

$$\exp\left\{-\left(\frac{x-g}{b}\right)^a\right\}$$

for  $x \ge g$  and 0 if x < g, where a is the shape, b is the scale, g is the location parameter, and x is the value of a generalized Weibull random variable.

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307 Domain *q*: –8e+307 to 8e+307

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge g$ 

Range: 0 to 1

#### invweibull(a,b,p)

Description: the inverse cumulative Weibull distribution with shape a and scale b: if

weibull(a,b,x) = p, then invweibull(a,b,p) = x

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307

Domain p: 0 to 1

Range: 1e-323 to 8e+307

#### invweibull(a,b,g,p)

Description: the inverse cumulative Weibull distribution with shape a, scale b, and location g: if

weibull(a,b,g,x) = p, then invweibull(a,b,g,p) = x

Domain a: 1e-323 to 8e+307 Domain b: 1e-323 to 8e+307 Domain g: -8e+307 to 8e+307

Domain p: 0 to 1

Range: q + c(epsdouble) to 8e+307

#### invweibulltail(a,b,p)

Description: the inverse reverse cumulative Weibull distribution with shape a and scale b: if

weibulltail(a,b,x) = p, then invweibulltail(a,b,p) = x

Domain a: 1e-323 to 8e+307 Domain b: 1e-323 to 8e+307

Domain p: 0 to 1

Range: 1e-323 to 8e+307

### invweibulltail(a,b,g,p)

Description: the inverse reverse cumulative Weibull distribution with shape a, scale b, and location

g: if weibulltail(a,b,g,x) = p, then

invweibulltail(a,b,g,p) = x

1e-323 to 8e+307 Domain a: Domain b: 1e-323 to 8e+307 Domain g: -8e+307 to 8e+307

Domain p: 0 to 1

Range: q + c(epsdouble) to 8e+307

# Weibull (proportional hazards) distribution

#### weibullphden(a,b,x)

Description: the probability density function of the Weibull (proportional hazards) distribution

with shape a and scale b

weibullphden(a,b,x) = weibullphden(a,b,0,x), where a is the shape, b is the scale, and x is the value of Weibull (proportional hazards) random variable.

Domain a: 1e-323 to 8e+307 Domain b: 1e-323 to 8e+307 1e-323 to 8e+307 Domain x: 0 to 8e+307 Range:

#### weibullphden(a,b,g,x)

Description: the probability density function of the Weibull (proportional hazards) distribution with shape a, scale b, and location g

> The probability density function of the Weibull (proportional hazards) distribution is defined as

$$ba(x-g)^{a-1}\exp\{-b(x-g)^a\}$$

for  $x \ge g$  and 0 for x < g, where a is the shape, b is the scale, g is the location parameter, and x is the value of a Weibull (proportional hazards) random variable.

Domain a: 1e-323 to 8e+307 Domain b: 1e-323 to 8e+307 Domain g: -8e+307 to 8e+307

-8e+307 to 8e+307; interesting domain is  $x \ge g$ Domain x:

Range: 0 to 8e + 307

#### weibullph(a,b,x)

Description: the cumulative Weibull (proportional hazards) distribution with shape a and scale b

weibullph(a,b,x) = weibullph(a,b,0,x), where a is the shape, b is the scale, and x is the value of Weibull random variable.

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307 Domain *x*: 1e–323 to 8e+307

Range: 0 to 1

# weibullph(a,b,g,x)

Description: the cumulative Weibull (proportional hazards) distribution with shape a, scale b, and location q

The cumulative Weibull (proportional hazards) distribution is defined as

$$1 - \exp\left\{-b(x-g)^a\right\}$$

for  $x \ge g$  and 0 if x < g, where a is the shape, b is the scale, g is the location parameter, and x is the value of a Weibull (proportional hazards) random variable. The mean of the Weibull (proportional hazards) distribution is

$$g + b^{-\frac{1}{a}} \Gamma\{(a+1)/a)\}$$

and its variance is

$$b^{-\frac{2}{a}} \left( \Gamma\{(a+2)/a\} - [\Gamma\{(a+1)/a\}]^2 \right)$$

where  $\Gamma()$  is the gamma function described in  $\operatorname{Ingamma}(x)$ .

Domain a: 1e–323 to 8e+307 Domain b: 1e–323 to 8e+307 Domain g: —8e+307 to 8e+307

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge g$ 

Range: 0 to 1

#### weibullphtail(a,b,x)

Description: the reverse cumulative Weibull (proportional hazards) distribution with shape a and scale b

weibullphtail(a,b,x) = weibullphtail(a,b,0,x), where a is the shape, b is the scale, and x is the value of a Weibull (proportional hazards) random variable.

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307 Domain *x*: 1e–323 to 8e+307

### weibullphtail(a,b,g,x)

Description: the reverse cumulative Weibull (proportional hazards) distribution with shape a, scale

b, and location g

The reverse cumulative Weibull (proportional hazards) distribution is defined as

$$\exp\left\{-b(x-g)^a\right\}$$

for  $x \ge g$  and 0 of x < g, where a is the shape, b is the scale, g is the location parameter, and x is the value of a Weibull (proportional hazards) random variable.

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307 Domain *g*: –8e+307 to 8e+307

Domain x: -8e+307 to 8e+307; interesting domain is  $x \ge g$ 

Range: 0 to 1

#### invweibullph(a,b,p)

Description: the inverse cumulative Weibull (proportional hazards) distribution with shape a and

scale b: if weibullph(a,b,x) = p, then invweibullph(a,b,p) = x

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307

Domain p: 0 to 1

Range: 1e-323 to 8e+307

# invweibullph(a,b,g,p)

Description: the inverse cumulative Weibull (proportional hazards) distribution with shape a, scale b,

and location g: if weibullph(a,b,g,x) = p, then invweibullph(a,b,g,p) = x

Domain a: 1e–323 to 8e+307 Domain b: 1e–323 to 8e+307 Domain g: -8e+307 to 8e+307

Domain p: 0 to 1

Range: q + c(epsdouble) to 8e+307

#### invweibullphtail(a,b,p)

Description: the inverse reverse cumulative Weibull (proportional hazards) distribution with shape a

and scale b: if weibullphtail(a,b,x) = p, then invweibullphtail(a,b,p) = x

Domain a: 1e–323 to 8e+307 Domain b: 1e–323 to 8e+307

Domain p: 0 to 1

Range: 1e-323 to 8e+307

#### invweibullphtail(a,b,g,p)

Description: the inverse reverse cumulative Weibull (proportional hazards) distribution with shape

a, scale b, and location g: if weibullphtail(a, b, g, x) = p, then

invweibullphtail(a,b,q,p) = x

Domain *a*: 1e–323 to 8e+307 Domain *b*: 1e–323 to 8e+307 Domain *g*: –8e+307 to 8e+307

Domain p: 0 to 1

Range: g + c(epsdouble) to 8e+307

#### Wishart distribution

#### lnwishartden(df, V, X)

Description: the natural logarithm of the density of the Wishart distribution; missing if  $df \leq n-1$ 

df denotes the degrees of freedom, V is the scale matrix, and X is the Wishart

random matrix. Domain df: 1 to 1e+100 (may be nonintegral)

Domain V:  $n \times n$ , positive-definite, symmetric matrices Domain X:  $n \times n$ , positive-definite, symmetric matrices

Range: -8e+307 to 8e+307

#### lniwishartden(df, V, X)

Description: the natural logarithm of the density of the inverse Wishart distribution; missing if

 $df \le n-1$ 

 $\mathit{df}$  denotes the degrees of freedom, V is the scale matrix, and X is the inverse

Wishart random matrix.

Domain df: 1 to 1e+100 (may be nonintegral)

Domain V:  $n \times n$ , positive-definite, symmetric matrices Domain X:  $n \times n$ , positive-definite, symmetric matrices

Range: -8e+307 to 8e+307

John Wishart (1898–1956) was born in Montrose, Scotland. He obtained a degree in mathematics and physics from the University of Edinburgh. He learned mathematics from E. T. Whittaker, upon whose recommendation he became Karl Pearson's research assistant. During his apprenticeship, he worked on approximations to the incomplete beta function and published multiple papers on this topic. He is best known for deriving the generalized product moment distribution, which was consequently named the Wishart distribution. This distribution is a critical component in the calculation of covariance matrices and Bayesian statistics.

Wishart served in both world wars, fighting with the Black Watch regiment in the first and working for the Intelligence Corps in the second. Upon his return from World War II, he resumed his involvement with the Royal Statistical Society, becoming chairman of the Research Section in 1945. A few years later, he also served as Associate Editor for the journal *Biometrika*.

He taught courses in statistics and agriculture at Cambridge and became the Head of the Statistical Laboratory. He published multiple papers applying statistical methods to agricultural research and was involved with the United Nations Food and Agriculture Organization. He was in Mexico to establish an agricultural research center on behalf of this organization when he died.

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# Also see

- [FN] Functions by category
- [D] **egen** Extensions to generate
- [D] generate Create or change contents of variable
- [M-4] **Statistical** Statistical functions
- [U] 13.3 Functions

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