Title

bmastats jointness — Jointness measures for predictors after BMA regression

Description

bmastats jointness reports various jointness measures between pairs of predictors after the bmaregress command. These jointness measures assess the degree of inclusion dependency between two predictors across the models visited by bmaregress.

Quick start

Compute all available jointness measures for predictors x1 and x2

    bmastats jointness x1 x2

Compute the default Doppelhofer–Weeks jointness measure for predictors x1 through x4

    bmastats jointness x1-x4

Same as above, but compute Ley–Steel type 2 jointness measure

    bmastats jointness x1-x4, lsteel2

Same as above, but compute modified Yule's Q jointness measure

    bmastats jointness x1-x4, yuleqm frequency

Compute all available jointness measures

    bmastats jointness x1-x4, all

Menu

Statistics > Bayesian model averaging > Jointness measures

Syntax

> bmastats jointness *varlist* [, *jointopts* frequency]

jointopts	Description
all	all available jointness measures; default with two predictors
dweeks	Doppelhofer–Weeks measure; default with more than two predictors
lsteel1	Ley–Steel type 1 measure
lsteel2	Ley–Steel type 2 measure
yuleq	Yule's Q measure
yuleqm	modified Yule's Q measure; available only with sampling and requires option frequency
ls1	synonym for lsteel1
ls2	synonym for lsteel2
yq	synonym for yuleq
yqm	synonym for yuleqm

collect is allowed; see [U] **11.1.10 Prefix commands**.

Options

> [Main]

all computes and reports all available jointness measures. This is the default for two predictors. If more than two predictors are specified, one table for each measure is reported.

dweeks computes and reports the Doppelhofer–Weeks measure. This is the default when more than two predictors are specified. dw is a synonym for dweeks.

lsteel1 computes and reports the Ley–Steel type 1 measure. ls1 is a synonym for lsteel1.

lsteel2 computes and reports the Ley–Steel type 2 measure. ls2 is a synonym for lsteel2.

yuleq computes and reports Yule's Q measure. yq is a synonym for yuleq.

yuleqm computes and reports the modified Yule's Q measure. yqm is a synonym for yuleqm. This measure is available only with sampling and requires that you also specify option frequency.

frequency specifies that frequency estimates of posterior model probabilities (PMPs) based on Markov chain Monte Carlo be used in computations. These estimates are available only with sampling methods, when MCMC sample is available. That is, they are not available with model enumeration. With a fixed g, analytical PMPs are used in computations by default. With a random g, analytical PMPs are not available, and thus frequency PMPs are used (option frequency is implied).

Remarks and examples

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Remarks are presented under the following headings:

> *Jointness as a measure of variable-inclusion dependence*
> *Example: Jointness of growth determinants*

Jointness as a measure of variable-inclusion dependence

In Bayesian model averaging (BMA), marginal inferential measures such as posterior inclusions probabilities (PIPs) describe the importance of a single predictor in explaining the outcome after accounting for model uncertainty. However, joint inferential measures that explore the interrelations between predictors are often of interest too. For instance, we might want to know whether the importance of one predictor is affected by the presence of another across all the models. BMA provides various jointness measures to address this.

BMA defines jointness measures with respect to the joint posterior distribution of inclusion of predictors over the model space. Specifically, bivariate jointness measures are used to describe the inclusion patterns for pairs of predictors. That is, bivariate jointness assesses the tendency of two predictors to be included together or exclusively across the model space. In what follows, we will imply "bivariate jointness measures" when we say "jointness measures".

In general, jointness means that the two predictors are *complements*; that is, they add additional explanatory power for the outcome when included together. Thus, these predictors tend to be included together in a model with high PIP. Disjointness means that the two predictors are *substitutes*; that is, when one predictor is in a model, the inclusion of the other does not add any additional explanatory power for the outcome. These predictors tend to be excluded whenever the other one is in a model.

BMA literature introduces various jointness measures. The working paper by Doppelhofer and Weeks (2006) was the first to introduce a jointness measure in the context of BMA. Later, Doppelhofer and Weeks (2009) introduced a modified version of the original measure, which we refer to as the Doppelhofer–Weeks measure, or DW.

DW is defined as the log of the cross-product ratio of binary indicators of predictor inclusion. It corresponds to the log of the ratio of posterior odds of including one predictor given the other is included to that of including that predictor given the other is not included. DW takes values in the $(-\infty, \infty)$ interval. A positive DW indicates jointness between two predictors, and a negative DW indicates disjointness. DW is undefined whenever the PIP of either predictor is zero or one. The authors provide the following thresholds to interpret this measure:

Table 1. Thresholds for the DW measure

DW	Interpretation
$(-\infty, -2)$	Strong disjointness
$[-2, -1)$	Significant disjointness
$[-1, 1]$	Independent inclusion
$(1, 2]$	Significant jointness
$(2, \infty)$	Strong jointness

Ley and Steel (2007) introduce two other measures of jointness, the Ley–Steel type 1 measure (LS1) and the Ley–Steel type 2 measure (LS2). LS1 is defined as the ratio of the posterior probability of including both predictors to that of including either one. LS2 is defined as the ratio of the posterior probability of including both predictors to that of including either one, but not both. The latter corresponds to the posterior odds ratio of the models that include both predictors to the models that include these predictors individually. These measures take values on $[0, 1]$ and $[0, \infty)$, respectively. Unlike DW, LS1 and LS2 are defined even when the PIP of one of the predictors equals zero or one. For both measures, low values are interpreted as evidence of disjointness between the two predictors, and high values as evidence of jointness. The authors propose the following thresholds to interpret LS2 [LS1 can be determined based on $LS1 = LS2/(1 + LS2)$]:

Table 2. Thresholds for the LS2 and LS1

LS2	LS1*	Interpretation
$[0, 0.01)$	$[0, 0.01)$	Decisive disjointness
$[0.01, 0.03)$	$[0.01, 0.03)$	Very strong disjointness
$[0.03, 0.1)$	$[0.03, 0.09)$	Strong disjointness
$[0.1, 0.22)$	$[0.09, 0.18)$	Favorable disjointness
$[0.22, 3)$	$[0.18, 0.75)$	Independent inclusion
$[3, 10)$	$[0.75, 0.91)$	Favorable jointness
$[10, 30)$	$[0.91, 0.97)$	Strong jointness
$[30, 100)$	$[0.97, 0.99)$	Very strong jointness
$[100, \infty)$	$[0.99, 1]$	Decisive jointness

* rounded to 0.01 based on LS1 = LS2/(1 + LS2)

The Yule's Q binary similarity measure (YQ; for example, Yule [1912]) is also used as one of the jointness measures in BMA, where the inclusion of a predictor in a model is viewed as a binary event. This measure then compares the number of pairs in agreement (both predictors included or excluded) with the number of pairs in disagreement (only one of the predictors is included) relative to the total number of pairs. It takes values on $[-1, 1]$ and, as pointed out by Hofmarcher et al. (2018), can be viewed as a mapping of DW from $(-\infty, \infty)$ to $[-1, 1]$ with more interpretable bounds. Positive YQ implies jointness, negative YQ implies disjointness, and YQ close to zero means there is no association between the inclusion of the two predictors.

Like DW, YQ is undefined for predictors with PIPs equal to zero or one. To rectify this, in the case of sampling, Hofmarcher et al. (2018) proposed a modified Yule's Q measure (YQM), which can be viewed as a zero-cells adjustment in a cross-tabulation of the two binary inclusion indicators that adds 0.5 to zero cells (with a slight denominator adjustment). The rationale for this adjustment comes from assuming a multinomial distribution for the four counts in the cross-tabulation and a conjugate Dirichlet prior for the multinomial probabilities. An uninformative Jeffreys hyperprior is assumed for the parameters of the Dirichlet prior, which leads to the equal-cell adjustments of 0.5 for each cell. The interpretation of YQM is the same as for YQ.

There is no definitive recommendation in the literature for which measure should be preferred in practice. Hofmarcher et al. (2018) compare several theoretical properties of the five measures, and only YQM satisfies all properties considered by the authors. Unlike LS1, LS2, and YQM, the DW and YQ measures are undefined for predictors with PIPs of zero or one, but Man (2018) finds that DW produces more diverse results across various datasets, one of the desirable properties of patterns, compared with LS1 and LS2. The latter tend to produce more similar results across different datasets often favoring disjointness. Man (2018) concludes that jointness measures should be interpreted carefully in practice and that perhaps an aggregation of information from different measures might be a better approach than choosing just one measure.

Example: Jointness of growth determinants

In this section, we demonstrate the use of bmastats jointness to compute several jointness measures for the economic growth data considered by Hofmarcher et al. (2018) and Ley and Steel (2007), among others.

For a pair of predictors, bmastats jointness reports all available jointness measures. If more predictors are specified, the command reports the DW measure by default, but you can request other measures by specifying the respective options or use the all option to report all measures. The YQM measure is available only with sampling and only when the frequency option is also specified.

▷ Example 1: Jointness of growth determinants

Consider the econgrowth dataset from example 17 of [BMA] **bmaregress**, where a country's economic growth is modeled using 41 potential predictors, but here, to save time, we use a smaller MCMC size of 50,000.

```
. use https://www.stata-press.com/data/r18/econgrowth
(Economic growth data)

. bmaregress gdpgrowth abslat-yrsopen, mprior(uniform) mcmcsize(50000) rseed(18)

Burn-in ...
Simulation ...
Computing model probabilities ...

Bayesian model averaging                    No. of obs        =        72
Linear regression                           No. of predictors =        41
MC3 sampling                                          Groups  =        41
                                                      Always  =         0
                                            No. of models      =     6,394
                                                For CPMP >= .9 =     1,100
Priors:                                     Mean model size    =     9.422
  Models: Uniform                           Burn-in            =     2,500
   Cons.: Noninformative                    MCMC sample size   =    50,000
   Coef.: Zellner's g                       Acceptance rate    =    0.1720
       g: Benchmark, g = 1,681              Shrinkage, g/(1+g) =    0.9994
  sigma2: Noninformative                    Mean sigma2        =   .000055

Sampling correlation = 0.7884
```

gdpgrowth	Mean	Std. dev.	Group	PIP
gdp60	-.0162155	.0029558	16	.99968
confucian	.0563032	.0125301	9	.9993
lifeexp	.0008464	.0003022	22	.96505
equipinv	.1647531	.0609101	12	.95748
subsahara	-.0119874	.007829	38	.7794
muslim	.0087687	.0070261	24	.68554
ruleoflaw	.0087141	.0085463	35	.56117
ecoorg	.0014106	.0015073	10	.51286
protestants	-.0060471	.0070828	30	.48112
yrsopen	.0069856	.0080953	41	.4714
nequipinv	.0257697	.0321904	25	.43821
mining	.0163398	.0221579	23	.40574
latamerica	-.0013252	.0035112	21	.15603
prscenroll	.0032339	.0084244	31	.15337
buddha	.0015631	.0046289	6	.12457
blmktpm	-.0009528	.0028545	4	.12028
catholic	-.000449	.0026236	7	.092943
hindu	-.001704	.0068042	18	.073258
civllib	-.0001468	.0006411	8	.062838
rfexdist	-2.53e-06	.0000139	34	.042526
prexports	-.0004325	.0023676	29	.041549
polrights	-.0000654	.0003753	27	.039649
wardummy	-.0000986	.0007291	39	.026089
english	-.0001632	.0012182	11	.025993
age	-1.13e-06	8.39e-06	2	.024626
labforce	1.26e-09	1.29e-08	20	.023675
foreign	.0001093	.0008783	14	.021918
spanish	.0000587	.0007927	36	.016174
stdbmp	-2.46e-07	2.48e-06	37	.015815
french	.0000664	.0006795	15	.01551
ethnol	.0000719	.0008001	13	.013968
Always				
_cons	.0700867	.0203619	0	1

```
Note: Coefficient posterior means and std. dev. estimated from 6,394 models.
Note: Default prior is used for parameter g.
Note: 10 predictors with PIP less than .01 not shown.
```

Let's compute all available jointness measures for a pair of predictors, say, political rights, `polrights`, and civil liberties, `civllib`.

```
. bmastats jointness polrights civllib
Computing model probabilities ...
Variables: polrights civllib
```

	Jointness
Doppelhofer–Weeks	-2.994163
Ley–Steel type 1	.0013503
Ley–Steel type 2	.0013522
Yule's Q	-.9046195

```
Notes: Using analytical PMPs. See
       thresholds.
```

All reported jointness measures suggest disjointness or substitutability between these two predictors. Based on table 1, the DW value suggests strong disjointness ($-2.99 < -2$). Based on table 2, LS1 and LS2 values suggest decisive disjointness ($0.0014 < 0.01$). The YQ value of -0.9 is close to -1, which also suggests strong disjointness. The results suggest that the presence of one of `polrights` or `civllib` in the model greatly reduces the probability of inclusion of the other predictor.

From `bmaregress`, the initial GDP (`gdp60`), the fraction Confucian (`confucian`), and the life expectancy (`lifeexp`) are among the influential predictors of the economic growth. Let's examine their pairwise jointness by computing their DW scores first.

```
. bmastats jointness gdp60 confucian lifeexp, dweeks
Computing model probabilities ...
Doppelhofer–Weeks jointness
```

	gdp60	confucian	lifeexp
gdp60	.	.	7.073402
confucian	.	.	3.076793
lifeexp	7.073402	3.076793	.

```
Notes: Using analytical PMPs. See thresholds.
       The measure is undefined in some cases.
```

Variables `lifeexp` and `confucian` exhibit strong jointness ($DW = 3.08 > 2$) and thus can be viewed as complements. The same applies to predictors `lifeexp` and `gdp60`. But the measure is missing (undefined) for `gdp60` and `confucian`, which can happen as discussed in *Jointness as a measure of variable-inclusion dependence*.

Let's take a closer look at this pair of predictors:

```
. bmastats jointness confucian gdp60, dweeks
Computing model probabilities ...
Variables: confucian gdp60
```

	Jointness
Doppelhofer–Weeks	.

```
Notes: Using analytical PMPs. See
       thresholds. Some measures are
       undefined.
```

```
. display as txt "Prob. both excluded = " as res r(p00)
Prob. both excluded = 0

. display as txt "Prob. only gdp60 included = " as res r(p01)
Prob. only gdp60 included = .00070228

. display as txt "Prob. only confucian included = " as res r(p10)
Prob. only confucian included = .00031608

. display as txt "Prob. both included = " as res r(p11)
Prob. both included = .99898164
```

For a pair of predictors, bmastats jointness stores the respective joint probabilities of inclusion in
$r(p00)$, $r(p01)$, $r(p10)$, and $r(p11)$; see *Methods and formulas* for details. We can see that there
are no models in which both predictors are excluded, $r(p00) = 0$, and there are very few models
in which gdp60 is included but confucian is not, and vice versa. These results are not surprising
given that gdp60's PIP is almost 1. Given the high probability $r(p11) = 0.999$ for both predictors to
be included together, it might not be unreasonable to suspect jointness between these two predictors.

But let's explore other measures. Because the LS measures tend to provide similar results, let's
look at just one of them.

```
. bmastats jointness gdp60 confucian lifeexp, lsteel2

Computing model probabilities ...

Ley-Steel type 2 jointness
```

	gdp60	confucian	lifeexp
gdp60	.	980.9682	27.85594
confucian	980.9682	.	27.5357
lifeexp	27.85594	27.5357	.

```
Notes: Using analytical PMPs. See thresholds.
```

All three pairs of predictors exhibit strong jointness according to the LS2 measure.

Because of the similarity in the definitions of the DW and YQ measures, we would expect the latter
to be missing, too, for the confucian and gdp60 pair. We leave this for you to verify by specifying
the yuleq option with bmastats jointness.

Because sampling is used in this example, we can also compute the YQM measure. To do this, we
should also specify the frequency option to use frequency PMPs in the computation.

```
. bmastats jointness gdp60 confucian lifeexp, yuleqm frequency

Modified Yule's Q jointness
```

	gdp60	confucian	lifeexp
gdp60	.	-.9999963	.9843265
confucian	-.9999963	.	.5851771
lifeexp	.9843265	.5851771	.

```
Notes: Using frequency PMPs. See thresholds.
```

All values are above 0.5, which suggests strong jointness for all pairs of predictors.

It appears that gdp60, confucian, and lifeexp can be viewed as complements in describing
the economic growth, gdpgrowth, which is consistent with findings in Hofmarcher et al. (2018) and
Ley and Steel (2007).

◁

Stored results

bmastats jointness stores the following in r() with two predictors:

Scalars
r(dw)	Doppelhofer–Weeks measure
r(ls1)	Ley–Steel type 1 measure
r(ls2)	Ley–Steel type 2 measure
r(yq)	Yule's Q measure
r(yqm)	modified Yule's Q measure (when available)
r(p00)	posterior probability of excluding both predictors
r(p01)	posterior probability of including only the second predictor
r(p10)	posterior probability of including only the first predictor
r(p11)	posterior probability of including both predictors

Macros
r(varnames)	names of specified variables
r(pmptype)	analytical or frequency

bmastats jointness stores the following in r() with more than two predictors:

Macros
r(varnames)	names of specified variables
r(pmptype)	analytical or frequency

Matrices
r(dwmat)	Doppelhofer–Weeks measures
r(ls1mat)	Ley–Steel type 1 measures
r(ls2mat)	Ley–Steel type 2 measures
r(yqmat)	Yule's Q measures
r(yqmmat)	modified Yule's Q measures (when available)

Methods and formulas

Consider two predictors X_u and X_v of outcome $\mathbf{y}$. Let U denote the event that X_u is included in a model and V denote the event that X_v is included in a model. Let U^c and V^c be the respective complementary events. Let J be the full model space $J_F = \{1, 2, \ldots, 2^p\}$ in the case of model enumeration or the set of distinct models visited by the MCMC sampler in the case of sampling. The computation of jointness measures involves the following posterior probabilities:

$$P_U = \Pr(U|\mathbf{y}) = \sum_{j \in J} I(X_u \in M_j) P_a(M_j|\mathbf{y})$$

$$P_V = \Pr(V|\mathbf{y}) = \sum_{j \in J} I(X_v \in M_j) P_a(M_j|\mathbf{y})$$

$$P_{UV} = \Pr(U, V|\mathbf{y}) = \sum_{j \in J} I(X_u \in M_j, X_v \in M_j) P_a(M_j|\mathbf{y})$$

$$P_{U^c V} = \Pr(U^c, V|\mathbf{y}) = \sum_{j \in J} I(X_u \notin M_j, X_v \in M_j) P_a(M_j|\mathbf{y})$$

$$P_{UV^c} = \Pr(U, V^c|\mathbf{y}) = \sum_{j \in J} I(X_u \in M_j, X_v \notin M_j) P_a(M_j|\mathbf{y})$$

$$P_{U^c V^c} = \Pr(U^c, V^c|\mathbf{y}) = \sum_{j \in J} I(X_u \notin M_j, X_v \notin M_j) P_a(M_j|\mathbf{y})$$

where $I(\cdot)$ is the indicator function and $P_a(M_j|\mathbf{y})$ is the analytical PMP defined by (7) in *Posterior model probability* in *Methods and formulas* of [BMA] **bmaregress**. For a random g or with option `frequency`, the above posterior probabilities are estimated from an MCMC posterior sample of models $\{m_t\}_{t=1}^{T}$:

$$\widehat{P}_U = \frac{1}{T} \sum_{t=1}^{T} I(X_u \in m_t)$$

$$\widehat{P}_V = \frac{1}{T} \sum_{t=1}^{T} I(X_v \in m_t)$$

$$\widehat{P}_{UV} = \frac{1}{T} \sum_{t=1}^{T} I(X_u \in m_t, X_v \in m_t)$$

$$\widehat{P}_{U^cV} = \frac{1}{T} \sum_{t=1}^{T} I(X_u \notin m_t, X_v \in m_t)$$

$$\widehat{P}_{UV^c} = \frac{1}{T} \sum_{t=1}^{T} I(X_u \in m_t, X_v \notin m_t)$$

$$\widehat{P}_{U^cV^c} = \frac{1}{T} \sum_{t=1}^{T} I(X_u \notin m_t, X_v \notin m_t)$$

Doppelhofer and Weeks (2009) proposed the following measure of jointness:

$$J_{\mathrm{DW}} = \log\left(\frac{P_{UV} P_{U^cV^c}}{P_{U^cV} P_{UV^c}}\right) \in (-\infty, \infty)$$

Ley and Steel (2007) proposed the following measures of jointness:

$$J_{\mathrm{LS1}} = \frac{P_{UV}}{P_U + P_V - P_{UV}} \in [0, 1]$$

$$J_{\mathrm{LS2}} = \frac{P_{UV}}{P_{UV^c} + P_{U^cV}} \in [0, \infty)$$

The YQ measure (Yule 1912) is

$$J_{\mathrm{YQ}} = \frac{P_{UV} P_{U^cV^c} - P_{UV^c} P_{U^cV}}{P_{UV} P_{U^cV^c} + P_{UV^c} P_{U^cV}} \in [-1, 1]$$

When sampling is used, the posterior probabilities in the above formulas are replaced with their respective frequency-based estimates, such as $\widehat{P}_{UV}$ and $\widehat{P}_{U^cV}$. Also, when the `frequency` option is specified, the YQM measure (Hofmarcher et al. 2018) is computed as

$$J_{\mathrm{YQM}} = \frac{(\widehat{P}_{UV} + \xi)(\widehat{P}_{U^cV^c} + \xi) - (\widehat{P}_{UV^c} + \xi)(\widehat{P}_{U^cV} + \xi)}{(\widehat{P}_{UV} + \xi)(\widehat{P}_{U^cV^c} + \xi) + (\widehat{P}_{UV^c} + \xi)(\widehat{P}_{U^cV} + \xi) - 2\xi^2} \in [-1, 1]$$

where $\xi = 1/(2T)$.

References

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Also see

[BMA] **bmastats** — Summary for models and predictors after BMA regression

[BMA] **bmaregress** — Bayesian model averaging for linear regression

[BMA] **BMA postestimation** — Postestimation tools for Bayesian model averaging

[BMA] **Glossary**