meta me postestimation - Postestimation tools for multilevel mixed-effects meta-analysis

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Postestimation commands

meta meregress and meta multilevel allow the same postestimation commands. The following postestimation commands are of special interest after meta meregress and meta multilevel:

Command	Description	
estat group	summarize the composition of the nested groups	
estat heterogeneity	compute multilevel heterogeneity statistics	
estat recovariance	display the estimated random-effects covariance matrices	
estat sd	display variance components as standard deviations and correlations	

The following standard postestimation commands are also available:

Command	Description	
contrast	contrasts and ANOVA-style joint tests of estimates	
estat ic	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC)	
estat summarize	summary statistics for the estimation sample	
estat vce	variance-covariance matrix of the estimators (VCE)	
estimates	cataloging estimation results	
etable	table of estimation results	
lincom	point estimates, standard errors, testing, and inference for linear combina- tions of coefficients	
lrtest	likelihood-ratio test	
margins	marginal means, predictive margins, marginal effects, and average marginal effects	
marginsplot	graph the results from margins (profile plots, interaction plots, etc.)	
nlcom	point estimates, standard errors, testing, and inference for nonlinear com- binations of coefficients	
predict	predictions and their SEs, leverage statistics, etc.	
predictnl	point estimates, standard errors, testing, and inference for generalized predictions	
pwcompare	pairwise comparisons of estimates	
test	Wald tests of simple and composite linear hypotheses	
testnl	Wald tests of nonlinear hypotheses	

predict

Description for predict

predict creates a new variable containing predictions such as linear predictions, standard errors, fitted values, residuals, and standardized residuals. You can also obtain predictions of random effects and estimates of their standard errors.

Menu for predict

Statistics > Postestimation

Syntax for predict

Syntax for obtaining predictions other than best linear unbiased predictions (BLUPs) of random effects

predict [type] newvar [if] [in] [, statistic relevel(levelvar)]

Syntax for obtaining BLUPs of random effects and the BLUPs' standard errors

```
predict [type] { stub* | newvarlist } [if] [in], reffects [relevel(levelvar)
reses(resesspec)]
```

statistic	Description	
Main		
xb	linear prediction for the fixed portion of the model only; the default	
stdp	standard error of the fixed-portion linear prediction	
fitted	fitted values, fixed-portion linear prediction plus contributions based on predicted random effects	
<u>res</u> iduals	iduals residuals, response minus fitted values	
* <u>rsta</u> ndard	standardized residuals	

Unstarred statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when if e(sample) is not specified.

Options for predict

Main

xb, the default, calculates the linear prediction $\mathbf{X}_{j}\hat{\boldsymbol{\beta}}$ based on the estimated fixed effects (coefficients) in the model. This is equivalent to fixing all random effects in the model to their theoretical mean value of 0.

stdp calculates the standard error of the linear prediction $\mathbf{X}_{j}\widehat{\boldsymbol{\beta}}$.

fitted calculates fitted values, which are equal to the fixed-portion linear predictor *plus* contributions based on predicted random effects, $\mathbf{X}_j \hat{\boldsymbol{\beta}} + \mathbf{Z}_j \hat{\mathbf{u}}_j$. By default, the fitted values account for random effects from all levels in the model; however, if the relevel(*levelvar*) option is specified, then the fitted values are fit beginning with the topmost level down to and including level *levelvar*. For example, if trials are nested within regions, then typing

would produce region-level predictions. That is, the predictions would incorporate region-specific random effects but not those for each trial nested within each region.

- residuals calculates residuals, equal to the responses minus fitted values, $\hat{\theta}_j \mathbf{X}_j \hat{\beta} \mathbf{Z}_j \hat{\mathbf{u}}_j$. By default, the fitted values account for random effects from all levels in the model; however, if the relevel(*levelvar*) option is specified, then the fitted values are fit beginning at the topmost level down to and including level *levelvar*.
- rstandard calculates standardized residuals, equal to the residuals multiplied by the inverse square root of the estimated error covariance matrix.
- reffects calculates best linear unbiased predictions (BLUPs) of the random effects. By default, BLUPs for all random effects in the model are calculated. However, if the relevel(*levelvar*) option is specified, then BLUPs for only level *levelvar* in the model are calculated. For example, if trials are nested within regions, then typing

. predict b*, reffects relevel(region)

would produce BLUPs at the region level. You must specify q new variables, where q is the number of random-effects terms in the model (or level). However, it is much easier to just specify *stub** and let Stata name the variables *stub*1, *stub*2, ..., *stubq* for you.

- relevel (*levelvar*) specifies the level in the model at which predictions involving random effects are to be obtained; see the options above for the specifics. *levelvar* is the name of the variable describing the grouping at that level. This option is not allowed with statistic xb or stdp.
- reses(resesspec) calculates the standard errors of the random effects, where resesspec is

stub* | newvarlist [, comparative | diagnostic]

- comparative, the default, computes comparative random-effects standard errors. For linear multilevel models, these correspond to posterior standard deviations of random effects and to standard errors of marginal prediction errors $\hat{\mathbf{u}}_j - \mathbf{u}_j$. These standard errors are used for inference about the random effects.
- diagnostic computes diagnostic random-effects standard errors. These correspond to marginal standard errors of BLUPs, $SE(\widehat{\mathbf{u}}_i)$. These standard errors are used for model diagnostics.

By default, standard errors for all BLUPs in the model are calculated. However, if the relevel(*levelvar*) option is specified, then standard errors for only level *levelvar* in the model are calculated; see the reffects option.

You must specify q new variables, where q is the number of random-effects terms in the model (or level). However, it is much easier to just specify *stub** and let Stata name the variables *stub*1, *stub*2, ..., *stubq* for you. The new variables will have the same storage type as the corresponding random-effects variables.

The reffects and reses() options often generate multiple new variables at once. When this occurs, the random effects (or standard errors) contained in the generated variables correspond to the order in which the variance components are listed in the output of meta meregress or meta multilevel. Still, examining the variable labels of the generated variables (with the describe command, for instance) can be useful in deciphering which variables correspond to which terms in the model.

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margins

Description for margins

margins estimates margins of response for linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

margins [marginlist] [, options]			
<pre>margins [marginlist], predict(statistic) [options]</pre>			
statistic	Description		
xb	linear predictor for the fixed portion of the model only; the default		
stdp	not allowed with margins		
<u>fit</u> ted	not allowed with margins		
<u>res</u> iduals	not allowed with margins		
<u>rsta</u> ndard	not allowed with margins		
<u>ref</u> fects	not allowed with margins		

Statistics not allowed with margins are functions of stochastic quantities other than e(b).

For the full syntax, see [R] margins.

Remarks and examples

stata.com

Various predictions and statistics are available after fitting a multilevel meta-regression using meta multilevel or meta meregress. Below, we will discuss how to obtain BLUPs of the random effects. Random effects at different levels of hierarchy are not estimated when the model is fit but rather must be predicted after the estimation of the model parameters. The estimates of the random effects are in turn used to obtain other statistics such as the fitted values and residuals. These are useful for checking model assumptions and may be used in general as model-building tools.

Example 1: Obtaining predictions of random effects

In example 2 of [META] **meta meregress**, we conducted a multilevel meta-analysis to assess the effect of modifying the school calendar on students' achievement test scores. For completeness, we refit that model here:

. use https://www.stata-press.com/data/r18/schoolcal
(Effect of modified school calendar on student achievement)
. meta meregress stdmdiff || district: || school:, essevariable(se)
(output omitted)

The above model can also be fit by using the meta multilevel command as follows:

. meta multilevel stdmdiff, relevels(district school) essevariable(se)

We can use estat group to see how the data are broken down by district and school:

. estat group

Group variable	No. of	Observ	vations per	group
	groups	Minimum	Average	Maximum
district	11	3	5.1	11
school	56	1	1.0	1

We are reminded that each district had somewhere between 3 to 11 schools and that each school reported one effect size in our dataset.

Below, we predict the random effects using predict, reffects and obtain their diagnostic standard errors by specifying the reses(, diagnostic) option. Because we have two random effects in our model (at levels 2 and 3), we need to specify two new variable names with predict and two new variable names within reses(). Although it is typically much easier to specify a stubname, say, u*, where predict constructs variables u1 and u2 for you, here we will explicitly specify the names u3 and u2 for the variables corresponding to the random intercepts at level 3 and level 2, respectively. We will also specify se_u3 and se_u2 within the reses() option instead of a stubname u_se*. And we will use suboption diagnostic of the reses() option to request the diagnostic standard errors instead of the default comparative standard errors. The diagnostic standard errors are used for model diagnostics (Goldstein 2011; Skrondal and Rabe-Hesketh 2009).

- . predict double u3 u2, reffects reses(se_u3 se_u2, diagnostic)
- . by district, sort: generate tolist = (_n==1)

	district	u3	se_u3
1.	11	18998595	.07071818
5.	12	08467077	.13168501
9.	18	.1407273	.11790486
12.	27	.24064814	.13641505
16.	56	1072942	.13633364
20.	58	23650899	. 15003184
20. 31.	71	230500599	.12606073
34.	86	2004695	.1499012
42.	91	.05711692	.14284823
48.	108	14168396	.13094894
53.	644	01215679	.10054689

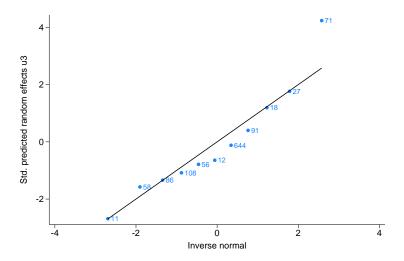
. list district u3 se_u3 if tolist

As an example, we listed the random-intercepts variable u3 with the corresponding standard error variable se_u3 for the M = 11 school districts. The purpose of variable tolist is to list only the unique values of u3 and se_u3 for each district. Had we not added the if tolist qualifier, row j (j = 1, ..., M) would have been repeated m_j times, where m_j is the number of schools within the *j*th district. The random intercepts are district-specific deviations from the overall mean effect size. For example, for district 18, the predicted standardized mean difference is 0.1407 higher than the overall effect size $\hat{b}_0 = 0.1847$, estimated in example 2 of [META] meta meregress, conditional on zero contribution from level-2 random intercepts.

Let's use the predicted random effects and their standard errors to compute a standardized randomeffects variable, ustan3, to check for outliers. This new variable corresponds to the standardized random effects at the district level (level 3). We will use the qnorm command (see [R] Diagnostic plots) to obtain the normal quantile plot.

```
. generate double ustan3 = u3/se_u3
```

- . label variable ustan3 "Std. predicted random effects u3"
- . qnorm ustan3 if tolist, mlabel(district) xtitle("Inverse normal")



From the plot, district 71 appears to be an outlier. Let's list the values for districts 71 and, for comparison, 27.

. list district school stdmdiff if inlist(district, 71, 27), separator(4)

	district	school	stdmdiff
12. 13. 14. 15.	27 27 27 27 27	1 2 3 4	.16 .65 .36 .6
31. 32. 33.	71 71 71	1 2 3	.3 .98 1.19

District 71 has 3 schools with students following the modified calendar scoring substantially higher $(\hat{\theta}_{71,1} = 0.3, \hat{\theta}_{71,2} = 0.98)$, and $\hat{\theta}_{71,3} = 1.19)$ compared with students from schools with modified calendars in other districts such as district 27.

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Example 2: Checking model fit

Continuing with example 1, we specify the fitted option to obtain the fitted values and plot them against the observed values of stdmdiff. By default, for the kth school within the jth district, the fitted values, $\hat{\beta}_0 + \hat{u}_j^{(3)} + \hat{u}_{jk}^{(2)}$, are computed based on random-effects contributions from all levels

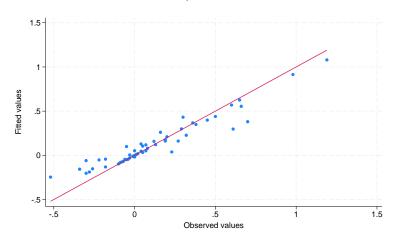
of hierarchy. Alternatively, we could specify the relevel(district) option to compute the fitted values, $\hat{\beta}_0 + \hat{u}_i^{(3)}$, based on contributions from level-3 random-effects only.

```
. predict double fit, fitted
```

```
. twoway (scatter fit stdmdiff)
```

```
(function y = x, range(stdmdiff)),
```

```
> legend(off) xtitle(Observed values) ytitle(Fitted values)
```



In the above code, we computed the fitted values using predict, fitted. We then produced a scatterplot of the fitted values versus the observed values of stdmdiff. We added a reference line y = x to assess goodness of fit. Studies that are close to the reference line have their fitted values close to the observed values. Overall, it seems that goodness of fit is satisfactory.

You could also use the rstandard option with predict to compute standardized residuals. In theory, the standardized residuals are useful for checking the normality assumption of the level-1 error terms.

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Methods and formulas

Continuing with the notation in *Methods and formulas* of [META] **meta meregress**, the three-level meta-regression model can be expressed compactly as

$$\hat{\boldsymbol{\theta}}_j = \mathbf{X}_j \boldsymbol{\beta} + \mathbf{Z}_j \mathbf{u}_j + \boldsymbol{\epsilon}_j, \quad j = 1, \dots, M$$

where $m_{j.} \times (q_3 + m_j q_2)$ matrix $\mathbf{Z}_j = (\mathbf{Z}_j^{(3)}, \mathbf{Z}_j^{(2)})$ and $(q_3 + m_j q_2) \times 1$ vector $\mathbf{u}_j = (\mathbf{u}_j^{(3)\prime}, \mathbf{u}_j^{(2)\prime})'$ with a $(q_3 + m_j q_2) \times (q_3 + m_j q_2)$ covariance matrix $\boldsymbol{\Sigma}_j$

$$\mathbf{\Sigma}_{j} = \operatorname{Var}\left(\mathbf{u}_{j}\right) = \begin{bmatrix} \mathbf{\Sigma}^{(\mathbf{3})} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{m_{j}} \otimes \mathbf{\Sigma}^{(2)} \end{bmatrix}$$

If we let $\mathbf{X} = (\mathbf{X}'_1, \mathbf{X}'_2, \dots, \mathbf{X}'_M)'$, $\mathbf{\Lambda} = \bigoplus_{j=1}^M \mathbf{\Lambda}_j$, $\mathbf{Z} = \bigoplus_{j=1}^M \mathbf{Z}_j$, and $\mathbf{u} = (\mathbf{u}'_1, \mathbf{u}'_2, \dots, \mathbf{u}'_M)'$, then the formulas used by predict for predicting random effects, residuals, etc. are described in *Methods and formulas* of [ME] **mixed postestimation** with $\mathbf{G} = \bigoplus_{j=1}^M \mathbf{\Sigma}_j$, $\mathbf{R} = \mathbf{\Lambda}$, and $\sigma_{\epsilon}^2 = 1$.

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When the reses() option is specified with reffects, the estimated covariance matrix of $\hat{\mathbf{u}}_j - \mathbf{u}_j$ is computed:

$$\widehat{\operatorname{Var}}\left(\widehat{\mathbf{u}}_{j}-\mathbf{u}_{j}\right)=\widehat{\boldsymbol{\Sigma}}-\widehat{\boldsymbol{\Sigma}}\mathbf{Z}_{j}'\mathbf{W}_{j}\left\{\left(\mathbf{W}_{j}\right)^{-1}-\mathbf{X}_{j}\operatorname{Var}(\widehat{\boldsymbol{\beta}})\mathbf{X}_{j}'\right\}\mathbf{W}_{j}\mathbf{Z}_{j}\widehat{\boldsymbol{\Sigma}}$$

The comparative standard errors of the random effects can be obtained by taking the square root of the diagonal elements of $\widehat{Var}(\widehat{\mathbf{u}}_j - \mathbf{u}_j)$.

If the diagnostic suboption is specified within reses(), then the estimated covariance matrix of $\hat{\mathbf{u}}_j$ is computed:

$$\widehat{\operatorname{Var}}\left(\widehat{\mathbf{u}}_{j}\right) = \widehat{\boldsymbol{\Sigma}}\mathbf{Z}_{j}'\mathbf{W}_{j}\left\{\left(\mathbf{W}_{j}\right)^{-1} - \mathbf{X}_{j}\operatorname{Var}(\widehat{\boldsymbol{\beta}})\mathbf{X}_{j}'\right\}\mathbf{W}_{j}\mathbf{Z}_{j}\widehat{\boldsymbol{\Sigma}}$$

The diagnostic standard errors of the random effects can be obtained by taking the square root of the diagonal elements of $\widehat{Var}(\widehat{\mathbf{u}}_i)$.

See Goldstein (2011), Skrondal and Rabe-Hesketh (2009), and Rabe-Hesketh and Skrondal (2022) for more details.

References

Goldstein, H. 2011. Multilevel Statistical Models. 4th ed. Chichester, UK: Wiley.

- Rabe-Hesketh, S., and A. Skrondal. 2022. *Multilevel and Longitudinal Modeling Using Stata.* 4th ed. College Station, TX: Stata Press.
- Skrondal, A., and S. Rabe-Hesketh. 2009. Prediction in multilevel generalized linear models. Journal of the Royal Statistical Society, Series A 172: 659–687. https://doi.org/10.1111/j.1467-985X.2009.00587.x.

Also see

- [META] meta meregress Multilevel mixed-effects meta-regression
- [META] meta multilevel Multilevel random-intercepts meta-regression
- [META] meta Introduction to meta
- [META] Glossary
- [META] Intro Introduction to meta-analysis
- [U] 20 Estimation and postestimation commands

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