lasso examples — Examples of lasso for prediction

Description Remarks and examples References Also see

Description

This entry contains more examples of lasso for prediction. It assumes you have already read [LASSO] **Lasso intro** and [LASSO] **lasso**.

Remarks and examples

Remarks are presented under the following headings:

Overview

In the examples of this entry, we use a dataset of a realistic size for lasso. It has 1,058 observations and 172 variables. Still, it is a little on the small side for lasso. Certainly, you can use lasso on datasets of this size, but lasso can also be used with datasets that have thousands or tens of thousands of variables.

The number of variables can even be greater than the number of observations. What is essential for lasso is that the set of potential variables contains a subset of variables that are in the true model (or something close to it) or are correlated with the variables in the true model.

As to how many variables there can be in the true model, we can say that the number cannot be greater than something proportional to $\sqrt{N}/\ln q$, where N is the number of observations, p is the number of potential variables, and $q = \max\{N, p\}$. We cannot, however, say what the constant of proportionality is. That this upper bound decreases with q can be viewed as the cost of performing covariate selection.

Using vl to manage variables

We will show how to use commands in the `vl` system to manage large numbers of variables. `vl` stands for "variable lists". The idea behind it is that we might want to run a lasso with hundreds or thousands or tens of thousands of variables specified as potential variables. We do not want to have to type all these variable names.

Many times, we will have a mix of different types of variables. Some we want to treat as continuous. Some we want to treat as categorical and use factor-variable operators with them to create indicator variables for their categories. See [U] 11.4.3 Factor variables.

The first goal of the vl system is to help us separate variables we want to treat as categorical from those we want to treat as continuous. The second goal of the system is to help us create named variable lists we can use as arguments to lasso or any other Stata command simply by referring to their names.

The purpose here is to illustrate the power of vl, not to explain in detail how it works or show all of its features. For that, see [D] vl.

We load the dataset we will use in these examples.

```
. use https://www.stata-press.com/data/r18/fakesurvey
(Fictitious survey data)
```

It is simulated data designed to mimic survey data. It has 1,058 observations and 172 variables.

```
. describe

Contains data from https://www.stata-press.com/data/r18/fakesurvey.dta
 Observations:         1,058                  Fictitious survey data
    Variables:           172                  14 Jun 2022 15:31
-------------------------------------------------------------------------------
Variable      Storage   Display    Value
    name         type    format    label      Variable label
-------------------------------------------------------------------------------
id              str8    %9s                   Respondent ID
gender          byte    %8.0g      gender     Gender
age             byte    %8.0g                 Age (y)
q1              byte    %10.0g                Question 1
q2              byte    %8.0g                 Question 2
q3              byte    %8.0g      yesno      Question 3
  (output omitted)
q160            byte    %8.0g      yesno      Question 160
q161            byte    %8.0g      yesno      Question 161
check8          byte    %8.0g                 Check 8
-------------------------------------------------------------------------------
Sorted by: id
```

The variables are a mix. Some we know are integer-valued scales that we want to treat as continuous variables in our models. There are a lot of 0/1 variables, and there are some with only a few categories that we will want to turn into indicator variables. There are some with more categories that we do not yet know whether to treat as categorical or continuous.

The first vl subcommand we run is vl set. Nonnegative integer-valued variables are candidates for use as factor variables. Because factor variables cannot be negative, any variable with negative values is classified as continuous. Any variable with noninteger values is also classified as continuous.

vl set has two options, categorical(#) and uncertain(#), that allow us to separate out the nonnegative integer-valued variables into three named variable lists: vlcategorical, vluncertain, and vlcontinuous.

When the number of levels (distinct values), L, is

$$2 \leq L \leq \texttt{categorical(\#)}$$

the variable goes in vlcategorical. When

$$\texttt{categorical(\#)} < L \leq \texttt{uncertain(\#)}$$

the variable goes in vluncertain. When

$$L > \texttt{uncertain(\#)}$$

the variable goes in vlcontinuous.

The defaults are categorical(10) and uncertain(100). For our data, we do not like the defaults, so we change them. We specify categorical(4) and uncertain(19). We also specify the option dummy to create a variable list, vldummy, consisting solely of 0/1 variables. Let's run vl set with these options.

```
. vl set, categorical(4) uncertain(19) dummy
```

Macro	Macro's contents # Vars	Description
System		
$vldummy	99	0/1 variables
$vlcategorical	16	categorical variables
$vlcontinuous	20	continuous variables
$vluncertain	27	perhaps continuous, perhaps categorical variables
$vlother	9	all missing or constant variables

Notes

1. Review contents of **vlcategorical** and **vlcontinuous** to ensure they are correct. Type **vl list vlcategorical** and type **vl list vlcontinuous**.
2. If there are any variables in **vluncertain**, you can reallocate them to **vlcategorical**, **vlcontinuous**, or **vlother**. Type **vl list vluncertain**.
3. Use **vl move** to move variables among classifications. For example, type **vl move (x50 x80) vlcontinuous** to move variables **x50** and **x80** to the continuous classification.
4. *vlnames* are global macros. Type the *vlname* without the leading dollar sign (**$**) when using **vl** commands. Example: **vlcategorical** *not* **$vlcategorical**. Type the dollar sign with other Stata commands to get a *varlist*.

The `vluncertain` variable list contains all the variables we are not sure whether we want to treat as categorical or continuous. We use `vl list` to list the variables in `vluncertain`.

```
. vl list vluncertain
```

Variable	Macro	Values	Levels
q12	$vluncertain	integers >=0	5
q18	$vluncertain	integers >=0	7
q23	$vluncertain	integers >=0	10
q27	$vluncertain	integers >=0	8
q28	$vluncertain	integers >=0	15
q35	$vluncertain	integers >=0	7
q39	$vluncertain	integers >=0	5
q54	$vluncertain	integers >=0	10
q63	$vluncertain	integers >=0	7
q66	$vluncertain	integers >=0	5
q80	$vluncertain	integers >=0	5
q81	$vluncertain	integers >=0	5
q92	$vluncertain	integers >=0	5
q93	$vluncertain	integers >=0	7
q99	$vluncertain	integers >=0	5
q103	$vluncertain	integers >=0	7
q107	$vluncertain	integers >=0	18
q111	$vluncertain	integers >=0	7
q112	$vluncertain	integers >=0	7
q119	$vluncertain	integers >=0	8
q120	$vluncertain	integers >=0	7
q124	$vluncertain	integers >=0	14
q127	$vluncertain	integers >=0	5
q132	$vluncertain	integers >=0	7
q135	$vluncertain	integers >=0	10
q141	$vluncertain	integers >=0	12
q157	$vluncertain	integers >=0	7

We are going to have to go through these variables one by one and reclassify them. We know we have several seven-level Likert scales in these data. We tabulate one of them.

```
. tabulate q18
```

Question 18	Freq.	Percent	Cum.
Very strongly disagree	139	13.15	13.15
Strongly disagree	150	14.19	27.34
Disagree	146	13.81	41.15
Neither agree nor disagree	146	13.81	54.97
Agree	174	16.46	71.43
Strongly agree	146	13.81	85.24
Very strongly agree	156	14.76	100.00
Total	1,057	100.00	

We look at all the variables with seven levels, and they are all Likert scales. We want to treat them as continuous in our models, so we move them out of `vluncertain` and into `vlcontinuous`.

```
. vl move (q18 q35 q63 q93 q103 q111 q112 q120 q132 q157) vlcontinuous
note: 10 variables specified and 10 variables moved.
```

Macro	# Added/Removed
$vldummy	0
$vlcategorical	0
$vlcontinuous	10
$vluncertain	-10
$vlother	0

When variables are moved into a new `vl` system-defined variable list, they are automatically moved out of their current system-defined variable list.

In our examples, we have three variables we want to predict: `q104`, a continuous variable; `q106`, a 0/1 variable; and `q107`, a count variable. Because we are going to use the variables in `vlcategorical` and `vlcontinuous` as potential variables to select in our lassos, we do not want these dependent variables in these variable lists. We move them into `vlother`, which is intended as a place to put variables we do not want in our models.

```
. vl move (q104 q106 q107) vlother
note: 3 variables specified and 3 variables moved.
```

Macro	# Added/Removed
$vldummy	-1
$vlcategorical	0
$vlcontinuous	-1
$vluncertain	-1
$vlother	3

Notice the parentheses around the variable names when we used `vl move`. The rule for `vl` is to use parentheses around variable names and to not use parentheses for variable-list names.

The system-defined variable lists are good for a general division of variables. But we need further subdivision for our models. We have four demographic variables, which are all categorical, but we want them included in all lasso models. So we create a user-defined variable list containing these variables.

```
. vl create demographics = (gender q3 q4 q5)
note: $demographics initialized with 4 variables.
```

We want to convert the variables in `vldummy` and `vlcategorical` into indicator variables. We create a new variable list, `factors`, containing the union of these lists. Because we want to handle the variables in `demographics` separately, we remove them from `factors`.

```
. vl create factors = vldummy + vlcategorical
note: $factors initialized with 114 variables.

. vl modify factors = factors - demographics
note: 4 variables removed from $factors.
```

The `vl substitute` command allows us to apply factor-variable operators to a variable list. We turn the variables in `demographics` and `factors` into factor variables.

```
. vl substitute idemographics = i.demographics

. vl substitute ifactors = i.factors
```

We are done using vl and we save our dataset. One nice feature of vl is that the variable lists are saved with the data.

```
. label data "Fictitious survey data with vl"

. save fakesurvey_vl
file fakesurvey_vl.dta saved
```

We are now ready to run some lassos.

Using splitsample

Well, almost ready. We want to evaluate our lasso predictions on a sample that we did not use to fit the lasso. So we decide to randomly split our data into two samples of equal sizes. We will fit models on one, and we will use the other to test their predictions.

Let's load the version of our dataset that contains our variable lists. We first increase maxvar because we are going to create thousands of interactions in a later example.

```
. clear all

. set maxvar 10000

. use https://www.stata-press.com/data/r18/fakesurvey_vl
(Fictitious survey data with vl)
```

Variable lists are not automatically restored. We have to run vl rebuild to make them active.

```
. vl rebuild
Rebuilding vl macros ...
```

Macro	Macro's contents # Vars	Description
System		
$vldummy	98	0/1 variables
$vlcategorical	16	categorical variables
$vlcontinuous	29	continuous variables
$vluncertain	16	perhaps continuous, perhaps categorical variables
$vlother	12	all missing or constant variables
User		
$demographics	4	variables
$factors	110	variables
$idemographics		factor-variable list
$ifactors		factor-variable list

We now use splitsample to generate a variable indicating the two subsamples.

```
. set seed 1234

. splitsample, generate(sample) nsplit(2)

. label define svalues 1 "Training" 2 "Testing"

. label values sample svalues
```

Lasso linear models

When fitting our lasso model, we can now specify variables succinctly using our `vl` variable lists. Variable lists are really global macros—we bet you already guessed this. Listing them under the header "Macro" in `vl` output was a real tip-off, right? Because they are global macros, when we use them as arguments in commands, we put a `$` in front of them.

We put parentheses around `idemographics`. This notation means that we want to force these variables into the model regardless of whether lasso wants to select them. See *Syntax* in [LASSO] **lasso**.

We also set the random-number seed using the `rseed()` option so that we can reproduce our results.

We fit `lasso` on the first subsample.

```
. lasso linear q104 ($idemographics) $ifactors $vlcontinuous
> if sample == 1, rseed(1234)
10-fold cross-validation with 100 lambdas ...
Grid value 1:     lambda = .8978025   no. of nonzero coef. =   4
Folds: 1...5....10   CVF = 16.93341
  (output omitted)
Grid value 23:    lambda = .1159557   no. of nonzero coef. =  74
Folds: 1...5....10   CVF = 12.17933
... cross-validation complete ... minimum found

Lasso linear model                          No. of obs         =        458
                                            No. of covariates  =        277
Selection: Cross-validation                 No. of CV folds    =         10
```

ID	Description	lambda	No. of nonzero coef.	Out-of-sample R-squared	CV mean prediction error
1	first lambda	.8978025	4	0.0147	16.93341
18	lambda before	.1846342	42	0.2953	12.10991
* 19	selected lambda	.1682318	49	0.2968	12.08516
20	lambda after	.1532866	55	0.2964	12.09189
23	last lambda	.1159557	74	0.2913	12.17933

```
* lambda selected by cross-validation.
. estimates store linearcv
```

After the command finished, we used `estimates store` to store the results in memory so that we can later compare these results with those from other lassos. Note, however, that `estimates store` only saves them in memory. To save the results to disk, use

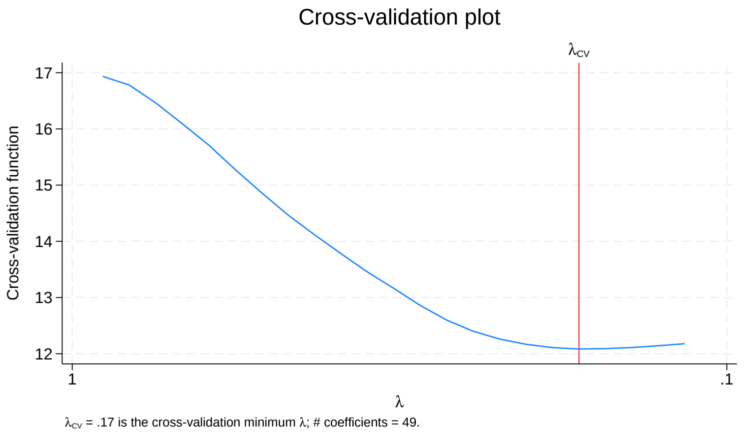
```
. estimates save filename
```

See [LASSO] **estimates store**.

The minimum of the cross-validation (CV) function was found to be at $\lambda = 0.1682318$. It selects λ^* as this λ, which corresponds to 49 variables in the model, out of 277 potential variables.

After fitting a lasso using CV to select λ, it is a good idea to plot the CV function and look at the shape of the curve around the minimum.

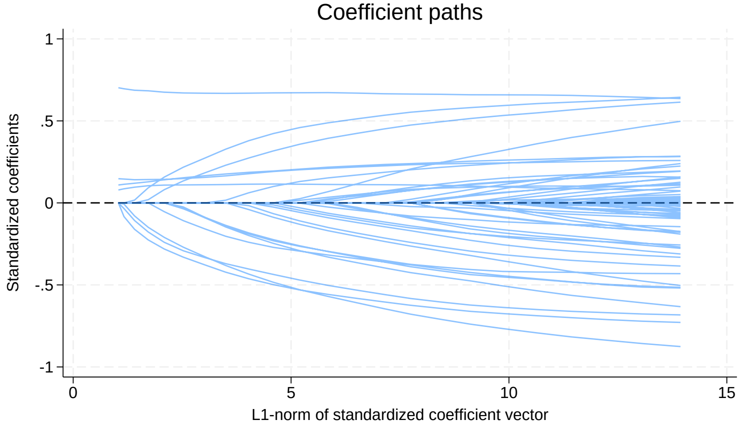
```
. cvplot
```



By default, the `lasso` command stops when it has identified a minimum. Computation time increases as λ's get smaller, so computing the CV function for smaller λ's is computationally expensive. We could specify the option `selection(cv, alllambdas)` to compute models for more small λ's. See [LASSO] **lasso** and [LASSO] **lasso fitting** for details and a description of less computationally intensive options to get more assurance that `lasso` has identified a minimum.

We can also get a plot of the size of the coefficients as they become nonzero and change as λ gets smaller. Typically, they get larger as λ gets smaller. But they can sometimes return to 0 after being nonzero.

```
. coefpath
```



We see four lines that do not start at 0. These are lines corresponding to the four variables in `idemographics` that we forced into the model.

Adaptive lasso

We are now going to run an adaptive lasso, which we do by specifying the option `selection(adaptive)`.

```
. lasso linear q104 ($idemographics) $ifactors $vlcontinuous
> if sample == 1, rseed(4321) selection(adaptive)
Lasso step 1 of 2:
10-fold cross-validation with 100 lambdas ...
Grid value 1:     lambda = .8978025   no. of nonzero coef. =   4
Folds: 1...5....10   CVF =   17.012
  (output omitted)
Grid value 24:    lambda = .1056545   no. of nonzero coef. =  78
Folds: 1...5....10   CVF = 12.40012
... cross-validation complete ... minimum found
Lasso step 2 of 2:
Evaluating up to 100 lambdas in grid ...
Grid value 1:     lambda = 48.55244   no. of nonzero coef. =  4
  (output omitted)
Grid value 100:   lambda = .0048552   no. of nonzero coef. = 59
10-fold cross-validation with 100 lambdas ...
Fold  1 of 10:  10....20....30....40....50....60....70....80....90....100
  (output omitted)
Fold 10 of 10:  10....20....30....40....50....60....70....80....90....100
... cross-validation complete
Lasso linear model                          No. of obs         =        458
                                            No. of covariates  =        277
Selection: Adaptive                         No. of lasso steps =          2
Final adaptive step results
```

ID	Description	lambda	No. of nonzero coef.	Out-of-sample R-squared	CV mean prediction error
25	first lambda	48.55244	4	0.0101	17.01083
77	lambda before	.3847698	46	0.3985	10.33691
* 78	selected lambda	.3505879	46	0.3987	10.33306
79	lambda after	.3194427	47	0.3985	10.33653
124	last lambda	.0048552	59	0.3677	10.86697

```
* lambda selected by cross-validation in final adaptive step.
. estimates store linearadaptive
```

Adaptive lasso performs multiple lassos. In the first lasso, a λ^* is selected, and penalty weights are constructed from the coefficient estimates. Then these weights are used in a second lasso, where another λ^* is selected. We did not specify how many lassos should be performed, so we got the default of two. We could specify more, but typically the selected λ^* does not change after the second lasso, or it changes little. See the `selection(adaptive)` option in [LASSO] **lasso**.

We can see details of the two lassos by using `lassoknots` and specifying the option `steps` to see all steps of the adaptive lasso.

```
. lassoknots, steps
```

Step	ID	lambda	No. of nonzero coef.	CV mean pred. error	Variables (A)dded, (R)emoved, or left (U)nchanged
1					
	1	.8978025	4	17.012	A 1.q3 1.q4 1.q5 1.gender
	2	.8180442	7	16.91096	A 0.q19 0.q85 3.q156
	3	.7453714	8	16.66328	A 0.q101
	4	.6791547	9	16.33224	A 0.q88
(output omitted)					
	23	.1159557	74	12.35715	A 3.q6 0.q40 0.q82 0.q98 0.q128 2.q134 0.q148 q157
	24	.1056545	78	12.40012	A 2.q6 0.q9 1.q34 4.q155
2					
	25	48.55244	4	17.01083	A 1.q3 1.q4 1.q5 1.gender
	26	44.23918	6	16.94087	A 0.q19 0.q85
(output omitted)					
	76	.4222844	45	10.33954	A 0.q44
	77	.3847698	46	10.33691	A q111
	* 78	.3505879	46	10.33306	U
	79	.3194427	47	10.33653	A 0.q97
	80	.2910643	48	10.3438	A 0.q138
(output omitted)					
	112	.0148272	59	10.7663	A q70
	124	.0048552	59	10.86697	U

```
* lambda selected by cross-validation in final adaptive step.
```

Notice how the scale of λ changes in the second lasso. That is because of the penalty weights generated by the first lasso.

The ordinary lasso selected 49 variables, and the adaptive lasso selected 46. It is natural to ask how much these two groups of variables overlap. When the goal is prediction, however, we are not supposed to care about this. Ordinary lasso might select one variable, and adaptive lasso might instead select another that is highly correlated to it. So it is wrong to place importance on any particular variable selected or not selected. It is the group of variables selected as a whole that matters.

Still, we cannot resist looking, and the `lassocoef` command was designed especially for this purpose. We specify `lassocoef` with the option `sort(coef, standardized)`. This sorts the listing by the absolute values of the standardized coefficients with the largest displayed first. `lassocoef` can list different types of coefficients and display them in different orderings. See [LASSO] **lassocoef**.

```
. lassocoef linearcv linearadaptive, sort(coef, standardized)

─────────────────────────────────────────
             │  linearcv  linearadaptive
─────────────┼───────────────────────────
         q19 │
          No │      x           x
             │
         q85 │
          No │      x           x
             │
          q5 │
         Yes │      x           x
             │
      3.q156 │      x           x
             │
        q101 │
          No │      x           x
```

(*output omitted*)

```
        q160 │
          No │      x           x
         age │      x           x
         q53 │      x           x
      2.q105 │      x
             │
        q102 │
          No │      x           x
             │
        q154 │
          No │      x           x
        q111 │      x           x
             │
        q142 │
          No │      x           x
       0.q55 │      x
       0.q97 │      x
             │
         q65 │
           4 │      x           x
             │
      1.q110 │      x           x
         q70 │      x
       _cons │      x           x
             │
         q44 │
          No │                  x
```

(*output omitted*)

```
─────────────────────────────────────────
Legend:
  b - base level
  e - empty cell
  o - omitted
  x - estimated
```

We see that the adaptive lasso did not select four variables that the lasso did, and it selected one that the lasso did not. All the differences occurred among the variables with smaller standardized coefficients.

The most important question to ask is which performed better for out-of-sample prediction. `lassogof` is the command for that. We specify the `over()` option with the name of our sample indicator variable, `sample`. We specify the `postselection` option because for linear models, postselection coefficients are theoretically slightly better for prediction than the penalized coefficients (which `lassogof` uses by default).

```
. lassogof linearcv linearadaptive, over(sample) postselection
Postselection coefficients

────────────────────────────┬──────────────────────────────────
Name                 sample │        MSE    R-squared        Obs
────────────────────────────┼──────────────────────────────────
linearcv                    │
                   Training │   8.652771       0.5065        503
                    Testing │   14.58354       0.2658        493
────────────────────────────┼──────────────────────────────────
linearadaptive              │
                   Training │   8.637575       0.5057        504
                    Testing │   14.70756       0.2595        494
────────────────────────────┴──────────────────────────────────
```

The ordinary lasso did a little better in this case than the adaptive lasso.

Cross-validation folds

CV works by dividing the data randomly into K folds. One fold is chosen, and then a linear regression is fit on the other $K-1$ folds using the variables in the model for that λ. Then using these new coefficient estimates, a prediction is computed for the data of the chosen fold. The mean squared error (MSE) of the prediction is computed. This process is repeated for the other $K-1$ folds. The K MSEs are then averaged to give the value of the CV function.

Let's increase the number of folds from the default of 10 to 20 by specifying `selection(cv, folds(20))`.

```
. lasso linear q104 ($idemographics) $ifactors $vlcontinuous
> if sample == 1, selection(cv, folds(20)) rseed(9999)
20-fold cross-validation with 100 lambdas ...
Grid value 1:     lambda = .8978025   no. of nonzero coef. =   4
Folds: 1...5....10....15....20   CVF = 17.08362
  (output omitted)
Grid value 23:    lambda = .1159557   no. of nonzero coef. =  74
Folds: 1...5....10....15....20   CVF = 12.12667
... cross-validation complete ... minimum found
Lasso linear model                          No. of obs        =        458
                                            No. of covariates =        277
Selection: Cross-validation                 No. of CV folds   =         20

──────────┬──────────────────────────────────────────────────────────────────
          │                                No. of      Out-of-      CV mean
          │                               nonzero       sample   prediction
       ID │     Description      lambda     coef.    R-squared        error
──────────┼──────────────────────────────────────────────────────────────────
        1 │    first lambda    .8978025         4       0.0059     17.08362
       19 │   lambda before    .1682318        49       0.2999     12.03169
     * 20 │ selected lambda    .1532866        55       0.3002     12.02673
       21 │    lambda after     .139669        62       0.2988     12.05007
       23 │     last lambda    .1159557        74       0.2944     12.12667
──────────┴──────────────────────────────────────────────────────────────────
* lambda selected by cross-validation.
. estimates store linearcv2
```

Which performs better for out-of-sample prediction?

```
. lassogof linearcv linearcv2, over(sample) postselection
Postselection coefficients
```

Name	sample	MSE	R-squared	Obs
linearcv				
	Training	8.652771	0.5065	503
	Testing	14.58354	0.2658	493
linearcv2				
	Training	8.545785	0.5126	502
	Testing	14.7507	0.2594	488

The first lasso with 10 folds did better than the lasso with 20 folds. This is generally true. More than 10 folds typically does not yield better predictions.

We should mention again that CV is a randomized procedure. Changing the random-number seed can result in a different λ^* being selected and so give different predictions.

BIC

We are now going to select λ^* by minimizing the BIC function, which we do by specifying the option `selection(bic)`.

```
. lasso linear q104 ($idemographics) $ifactors $vlcontinuous
> if sample == 1, selection(bic)
Evaluating up to 100 lambdas in grid ...
Grid value 1:     lambda = .8978025   no. of nonzero coef. =    4
                  BIC = 2618.642
Grid value 2:     lambda = .8180442   no. of nonzero coef. =    7
                  BIC = 2630.961
Grid value 3:     lambda = .7453714   no. of nonzero coef. =    8
                  BIC = 2626.254
Grid value 4:     lambda = .6791547   no. of nonzero coef. =    9
                  BIC = 2619.727
Grid value 5:     lambda = .6188205   no. of nonzero coef. =   10
                  BIC = 2611.577
Grid value 6:     lambda = .5638462   no. of nonzero coef. =   13
                  BIC = 2614.155
Grid value 7:     lambda = .5137556   no. of nonzero coef. =   13
                  BIC = 2597.164
Grid value 8:     lambda =   .468115   no. of nonzero coef. =   14
                  BIC = 2588.189
Grid value 9:     lambda = .4265289   no. of nonzero coef. =   16
                  BIC = 2584.638
Grid value 10:    lambda = .3886373   no. of nonzero coef. =   18
                  BIC = 2580.891
Grid value 11:    lambda = .3541118   no. of nonzero coef. =   22
                  BIC = 2588.984
Grid value 12:    lambda = .3226535   no. of nonzero coef. =   26
                  BIC = 2596.792
Grid value 13:    lambda = .2939899   no. of nonzero coef. =   27
                  BIC = 2586.521
Grid value 14:    lambda = .2678726   no. of nonzero coef. =   28
                  BIC = 2578.211
Grid value 15:    lambda = .2440755   no. of nonzero coef. =   32
                  BIC = 2589.632
```

```
Grid value 16:     lambda = .2223925   no. of nonzero coef. =  35
                   BIC = 2593.753
Grid value 17:     lambda = .2026358   no. of nonzero coef. =  37
                   BIC = 2592.923
Grid value 18:     lambda = .1846342   no. of nonzero coef. =  42
                   BIC = 2609.975
Grid value 19:     lambda = .1682318   no. of nonzero coef. =  49
                   BIC = 2639.437
... selection BIC complete ... minimum found
Lasso linear model                          No. of obs        =        458
                                            No. of covariates =        277
Selection: Bayesian information criterion
```

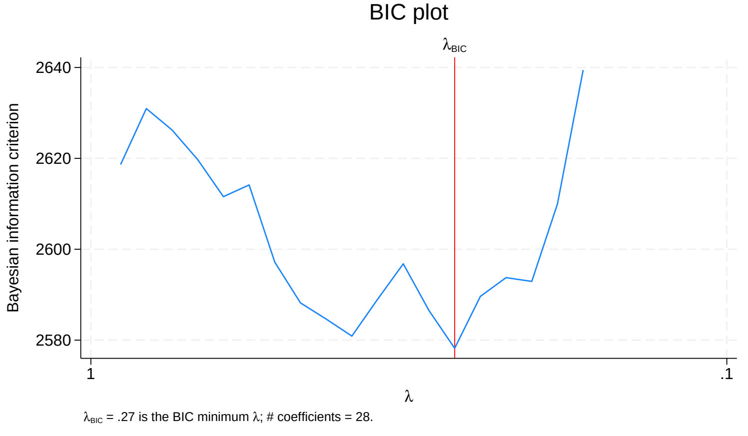
ID	Description	lambda	No. of nonzero coef.	In-sample R-squared	BIC
1	first lambda	.8978025	4	0.0308	2618.642
13	lambda before	.2939899	27	0.3357	2586.521
* 14	selected lambda	.2678726	28	0.3563	2578.211
15	lambda after	.2440755	32	0.3745	2589.632
19	last lambda	.1682318	49	0.4445	2639.437

```
* lambda selected by Bayesian information criterion.
. estimates store linearbic
```

The minimum of the BIC function was found to be at $\lambda = 0.268$. It selects λ^* as this λ, which corresponds to 28 variables in the model out of 277 potential variables.

After fitting a lasso using BIC, it is a good idea to plot the BIC function and look at the shape of the curve around the minimum.

```
. bicplot
```



We see that the BIC function rises sharply once it hits the minimum. By default, the `lasso` command stops when it has identified a minimum.

So far, we have fit lasso linear models using CV, an adaptive lasso, and BIC. Which one performs better in the out-of-sample prediction?

```
. lassogof linearcv linearadaptive linearbic, over(sample) postselection
Postselection coefficients
```

Name	sample	MSE	R-squared	Obs
linearcv				
	Training	8.652771	0.5065	503
	Testing	14.58354	0.2658	493
linearadaptive				
	Training	8.637575	0.5057	504
	Testing	14.70756	0.2595	494
linearbic				
	Training	9.740229	0.4421	508
	Testing	13.44496	0.3168	503

The BIC lasso performs the best.

More potential variables than observations

Lasso has no difficulty fitting models when the number of potential variables exceeds the number of observations.

We use `vl substitute` to create interactions of all of our factor-variable indicators with our continuous variables.

```
. vl substitute interact = i.factors##c.vlcontinuous
```

We fit the lasso.

```
. lasso linear q104 ($idemographics) $interact if sample == 1, rseed(1234)
note: 1.q32#c.q70 omitted because of collinearity with another variable.
note: 2.q34#c.q63 omitted because of collinearity with another variable.
  (output omitted)
10-fold cross-validation with 100 lambdas ...
Grid value 1:     lambda = 1.020288   no. of nonzero coef. =     4
Folds: 1...5....10   CVF = 16.93478
  (output omitted)
Grid value 34:    lambda = .2198144   no. of nonzero coef. =   106
Folds: 1...5....10   CVF = 12.91285
... cross-validation complete ... minimum found

Lasso linear model                         No. of obs         =        458
                                           No. of covariates  =      7,227
Selection: Cross-validation                No. of CV folds    =         10
```

ID	Description	lambda	No. of nonzero coef.	Out-of-sample R-squared	CV mean prediction error
1	first lambda	1.020288	4	0.0146	16.93478
29	lambda before	.2773743	80	0.2531	12.83525
* 30	selected lambda	.2647672	85	0.2545	12.81191
31	lambda after	.2527331	89	0.2541	12.81893
34	last lambda	.2198144	106	0.2486	12.91285

```
* lambda selected by cross-validation.
. estimates store big
```

There were 7,227 potential covariates in our model, of which lasso selected 85. That seems significantly more than the 49 selected by our earlier lasso.

Let's see how they do for out-of-sample prediction.

```
. lassogof linearcv big, over(sample) postselection
Postselection coefficients
```

Name	sample	MSE	R-squared	Obs
linearcv				
	Training	8.652771	0.5065	503
	Testing	14.58354	0.2658	493
big				
	Training	6.705183	0.6117	490
	Testing	17.00972	0.1403	478

Our model with thousands of potential covariates did better for in-sample prediction but significantly worse for out-of-sample prediction.

Factor variables in lasso

It is important to understand how lasso handles factor variables. Let's say we have a variable, `region`, that has four categories representing four different regions of the country. Other Stata estimation commands handle factor variables by setting one of the categories to be the base level; it then makes indicator variables for the other three categories, and they become covariates for the estimation.

Lasso does not set a base level. It creates indicator variables for all levels (`1.region`, `2.region`, `3.region`, and `4.region`) and adds these to the set of potential covariates. The reason for this should be clear. What if `1.region` versus the other three categories is all that matters for prediction? Lasso would select `1.region` and not select the other three indicators. If, however, `1.region` was set as a base level and omitted from the set of potential covariates, then lasso would have to select `2.region`, `3.region`, and `4.region` to pick up the `1.region` effect. It might be wasting extra penalty on three coefficients when only one was needed.

See [LASSO] **Collinear covariates**.

Lasso logit and probit models

`lasso` will also fit logit, probit, Poisson, and Cox models.

We fit a logit model.

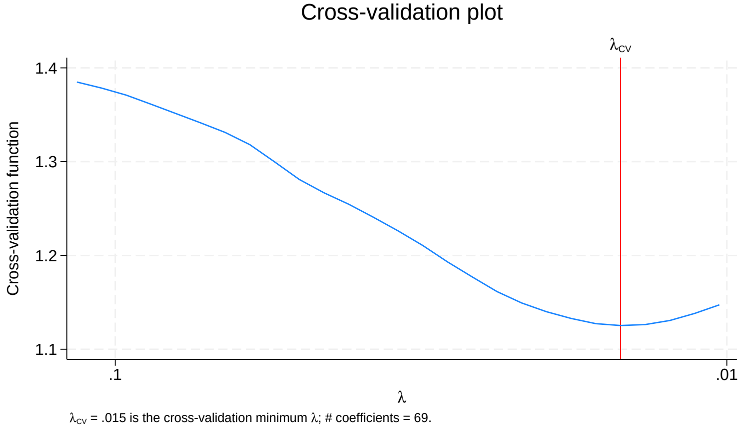
```
. lasso logit q106 $idemographics $ifactors $vlcontinuous
> if sample == 1, rseed(1234)
10-fold cross-validation with 100 lambdas ...
Grid value 1:     lambda = .1155342   no. of nonzero coef. =   0
Folds: 1...5....10   CVF = 1.384878
  (output omitted)
Grid value 27:    lambda =  .010285   no. of nonzero coef. =  88
Folds: 1...5....10   CVF = 1.147343
... cross-validation complete ... minimum found
Lasso logit model                          No. of obs        =        458
                                           No. of covariates =        277
Selection: Cross-validation                No. of CV folds   =         10
```

ID	Description	lambda	No. of nonzero coef.	Out-of-sample dev. ratio	CV mean deviance
1	first lambda	.1155342	0	-0.0004	1.384878
22	lambda before	.0163767	65	0.1857	1.127315
* 23	selected lambda	.0149218	69	0.1871	1.125331
24	lambda after	.0135962	73	0.1864	1.126333
27	last lambda	.010285	88	0.1712	1.147343

```
* lambda selected by cross-validation.
. estimates store logit
```

Logit and probit lasso models are famous for having CV functions that are more wiggly than those for linear models.

```
. cvplot
```



This curve is not as smoothly convex as was the CV function for the linear lasso shown earlier. But it is not as bad as some logit CV functions. Because the CV functions for nonlinear models are not as smooth, `lasso` has a stricter criterion for declaring that a minimum of the CV function is found than it has for linear models. `lasso` requires that five smaller λ's to the right of a nominal minimum be observed with larger CV function values by a relative difference of `cvtolerance(#)` or more. Linear models only require three such λ's be found before declaring a minimum and stopping.

Let's now fit a probit model.

```
. lasso probit q106 $idemographics $ifactors $vlcontinuous
> if sample == 1, rseed(1234)
10-fold cross-validation with 100 lambdas ...
Grid value 1:     lambda = .1844415   no. of nonzero coef. =   0
Folds: 1...5....10   CVF = 1.384877
  (output omitted)
Grid value 26:    lambda = .0180201   no. of nonzero coef. =  87
Folds: 1...5....10   CVF = 1.152188
... cross-validation complete ... minimum found
Lasso probit model                          No. of obs        =        458
                                            No. of covariates =        277
Selection: Cross-validation                 No. of CV folds   =         10
```

ID	Description	lambda	No. of nonzero coef.	Out-of-sample dev. ratio	CV mean deviance
1	first lambda	.1844415	0	-0.0004	1.384877
21	lambda before	.0286931	61	0.1820	1.132461
* 22	selected lambda	.0261441	64	0.1846	1.128895
23	lambda after	.0238215	70	0.1841	1.129499
26	last lambda	.0180201	87	0.1677	1.152188

```
* lambda selected by cross-validation.
. estimates store probit
```

`lassocoef` can be used to display coefficient values. Obviously, logit and probit coefficient values cannot be compared directly. But we do see similar relative scales.

```
. lassocoef logit probit, sort(coef, standardized) display(coef, standardized)

----------------------------------------
             |      logit      probit
-------------+--------------------------
        q142 |
          No |    -.50418   -.3065817
             |
        q154 |
          No |  -.3875702   -.2344515
             |
         q90 |
          No |  -.3771052   -.2288992
             |
          q8 |
          No |  -.3263827    -.200673
             |
  (output omitted)
             |
         q37 |
          No |  -.0128537   -.0062874
      2.q158 |   .0065661    .0012856
       3.q65 |  -.0062113
      3.q110 |  -.0055616
        q120 |   .0044864
      0.q146 |   -.004312
             |
         q95 |
           3 |   .0030261
----------------------------------------
Legend:
  b - base level
  e - empty cell
  o - omitted
```

The probit lasso selected five fewer variables than logit, and they were the five variables with the smallest absolute values of standardized coefficients.

We look at how they did for out-of-sample prediction.

```
. lassogof logit probit, over(sample)

Penalized coefficients

-------------------------------------------------------------
                        |                Deviance
Name             sample |   Deviance        ratio         Obs
------------------------+------------------------------------
logit                   |
               Training |    .8768969      0.3674         499
                Testing |    1.268346      0.0844         502
------------------------+------------------------------------
probit                  |
               Training |    .8833892      0.3627         500
                Testing |     1.27267      0.0812         503
-------------------------------------------------------------
```

Neither did very well. The out-of-sample deviance ratios were notably worse than the in-sample values. The deviance ratio for nonlinear models is analogous to R^2 for linear models. See *Methods and formulas* for [LASSO] **lassogof** for the formal definition.

We did not specify the `postselection` option in this case because there are no theoretical grounds for using postselection coefficients for prediction with nonlinear models.

Lasso Poisson models

Next, we fit a Poisson model.

```
. lasso poisson q107 $idemographics $ifactors $vlcontinuous
> if sample == 1, rseed(1234)
10-fold cross-validation with 100 lambdas ...
Grid value 1:     lambda = .5745539   no. of nonzero coef. =   0
Folds: 1...5....10   CVF = 2.049149
  (output omitted)
Grid value 21:    lambda =  .089382   no. of nonzero coef. =  66
Folds: 1...5....10   CVF = 1.653376
... cross-validation complete ... minimum found
Lasso Poisson model                          No. of obs        =        458
                                             No. of covariates =        277
Selection: Cross-validation                  No. of CV folds   =         10
```

ID	Description	lambda	No. of nonzero coef.	Out-of-sample dev. ratio	CV mean deviance
1	first lambda	.5745539	0	-0.0069	2.049149
16	lambda before	.1423214	37	0.1995	1.629222
* 17	selected lambda	.129678	45	0.1999	1.628315
18	lambda after	.1181577	48	0.1993	1.62962
21	last lambda	.089382	66	0.1876	1.653376

```
* lambda selected by cross-validation.
```

We see how it does for out-of-sample prediction.

```
. lassogof, over(sample)
Penalized coefficients
```

sample	Deviance	Deviance ratio	Obs
Training	1.289175	0.3515	510
Testing	1.547816	0.2480	502

Its in-sample and out-of-sample predictions are fairly close. Much closer than they were for the logit and probit models.

Lasso Cox models

`lasso` will also fit Cox proportional hazards models. We illustrate `lasso cox` with an example that predicts risk of death for stage I lung adenocarcinoma patients. Lung adenocarcinoma is one of the most common non-small-cell lung cancers.

Stage I adenocarcinoma indicates that the tumor size is relatively small, and cancer has not spread to other distant organs. Stage I adenocarcinoma patients usually have varied survival outcomes even though they are in the early cancer development stage. For example, Yu et al. (2016) show that, in one cohort, more than 50% of stage I adenocarcinoma patients died within 5 years after the initial diagnosis, while about 15% of the patients survived for more than 10 years.

Histopathology image features are indispensable for prognostic analysis. Examples of the histopathology image features include image granularity, image intensity, cell size and shape, pixel intensity of the cell, cell texture, area occupied by cells, neighboring relation of the cells, nucleus size and shape, and nucleus texture. We can use `lasso cox` to extract the top histopathology image features that distinguish short-term survivors from long-term survivors.

We have a fictitious survival dataset (`lungcancer.dta`) inspired by Yu et al. (2016). The variable `t` records either the time of death or censoring in months for stage I adenocarcinoma lung cancer patients. The indicator variable `died` is 1 or 0 if the patient died or is censored, respectively. There are 500 histopathology image features, `histfeature1` to `hisfeature500`, and only 250 patients. The analysis aims to classify a new patient into a low-risk or high-risk group, given the histopathology image features.

We first load the dataset and then type `stset` to show it has already been `stset`.

```
. use https://www.stata-press.com/data/r18/lungcancer
(Fictitious data on stage I adenocarcinoma lung cancer)

. stset
-> stset t, failure(died)

Survival-time data settings

         Failure event: died!=0 & died<.
Observed time interval: (0, t]
     Exit on or before: failure

--------------------------------------------------------------------------
        250  total observations
          0  exclusions
--------------------------------------------------------------------------
        250  observations remaining, representing
        211  failures in single-record/single-failure data
 18,465.093  total analysis time at risk and under observation
                                                At risk from t =         0
                                     Earliest observed entry t =         0
                                          Last observed exit t =       260
```

Next, we need to split the entire sample into training and testing data. The training data will be used for estimation, and the testing data will be used to measure the prediction performance. These steps are typically used in the microarray survival literature; for an application to the performance of a Cox model with lasso, see Sohn et al. (2009).

We use `splitsample` to split the data into two parts. The `generate(group)` option creates a new variable `group` for the identification of the training and testing data. That is, `group` equals 1 if it belongs to the training data or 0 if it belongs to the testing data. The `split(0.6 0.4)` option specifies that 60% of the entire data be used as training data and 40% of them be used as testing data. To make the results reproducible, we specify the `rseed()` option.

```
. splitsample, generate(group) split(0.6 0.4) rseed(12345)
```

For the convenience of later use, we separately save the training data (`lungcancer_training.dta`) and the testing data (`lungcancer_testing.dta`).

```
. preserve

. keep if group == 1
(100 observations deleted)

. save lungcancer_training
file lungcancer_training.dta saved

. restore

. preserve
```

```
. keep if group == 2
(150 observations deleted)

. save lungcancer_testing
file lungcancer_testing.dta saved

. restore
```

We are now ready to fit a `lasso cox` model using only the training data. By default, we use cross-validation. We specify `rseed()` to make the results reproducible.

```
. use lungcancer_training, clear
(Fictitious data on stage I adenocarcinoma lung cancer)

. lasso cox histfeature*, rseed(12345671)

        Failure _d: died
  Analysis time _t: t

10-fold cross-validation with 100 lambdas ...
Grid value 1:     lambda = .3539123   no. of nonzero coef. =   0
Folds: 1...5....10   CVF = 8.922501
Grid value 2:     lambda = .3378265   no. of nonzero coef. =   1
Folds: 1...5....10   CVF = 8.917438
  (output omitted)
Grid value 30:    lambda = .0918411   no. of nonzero coef. =  45
Folds: 1...5....10   CVF = 8.042941
Grid value 31:    lambda = .0876668   no. of nonzero coef. =  48
Folds: 1...5....10   CVF = 8.039609
Grid value 32:    lambda = .0836822   no. of nonzero coef. =  52
Folds: 1...5....10   CVF =  8.05246
Grid value 33:    lambda = .0798787   no. of nonzero coef. =  57
Folds: 1...5....10   CVF = 8.070293
Grid value 34:    lambda = .0762481   no. of nonzero coef. =  63
Folds: 1...5....10   CVF = 8.105045
... cross-validation complete ... minimum found

Lasso Cox model                          No. of obs        =        150
                                         No. of covariates =        500
Selection: Cross-validation              No. of CV folds   =         10
```

ID	Description	lambda	No. of nonzero coef.	In-sample dev. ratio	CV mean deviance
1	first lambda	.3539123	0	0.0000	8.922501
30	lambda before	.0918411	45	0.2199	8.042941
* 31	selected lambda	.0876668	48	0.2306	8.039609
32	lambda after	.0836822	52	0.2419	8.05246
34	last lambda	.0762481	63	0.2662	8.105045

```
* lambda selected by cross-validation.
```

`lasso cox` selects 48 of the 500 features. We can now predict the relative-hazard ratio, which we will call `riskscore_training`, and evaluate risk scores. We will use the median of `riskscore_training` as a threshold to classify a patient as low risk or high risk. We store the median value in a global macro (`median`) for later use.

```
. predict riskscore_training
(options hr penalized assumed; predicted hazard ratio with penalized
coefficients)
```

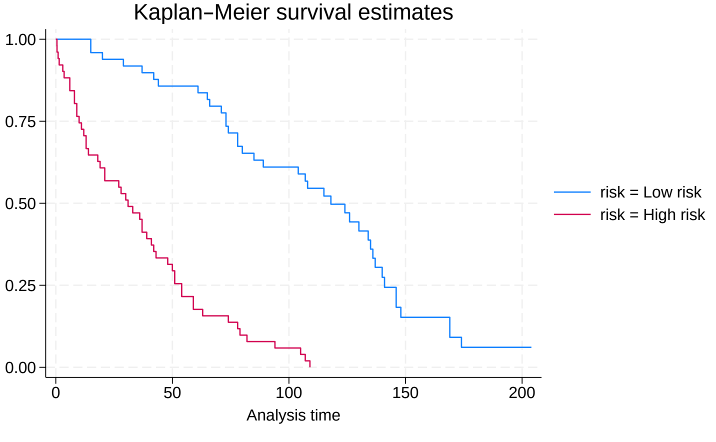
```
. summarize riskscore_training, detail

                 Predicted hazard ratio, penalized
-------------------------------------------------------------
      Percentiles      Smallest
 1%      .054982       .0414753
 5%     .0838301        .054982
10%     .1308778       .0702972       Obs                 150
25%     .3676802       .0727958       Sum of wgt.         150

50%     .9458244                      Mean           1.998198
                        Largest       Std. dev.       3.75226
75%     2.368032       9.962103
90%     4.912702       11.13334       Variance       14.07945
95%     6.651043        12.4411       Skewness       7.054249
99%      12.4411       39.40631       Kurtosis       67.68195

. global median = r(p50)
```

Based on the median of the predicted risk ratio in the training data, we now use the testing data to validate the model. First, we predict the risk ratio in the testing sample, which we will call `riskscore_testing`. Then, we compare `riskscore_testing` with the median of the risk ratio obtained in the training data (`$median`). If the predicted risk score is greater than or equal to the median, the patient is labeled as high risk. If the predicted risk score is less than the median, the patient is classified as low risk.

```
. use lungcancer_testing, clear
(Fictitious data on stage I adenocarcinoma lung cancer)

. predict riskscore_testing
(options hr penalized assumed; predicted hazard ratio with penalized
coefficients)

. generate byte risk = (riskscore_testing >= $median)

. label define risk_lb 1 "High risk" 0 "Low risk"

. label values risk risk_lb
```

To evaluate the effectiveness of risk classification, we first look at the Kaplan–Meier plot, which draws the survival curve for both low-risk and high-risk groups.

```
. sts graph, by(risk)

        Failure _d: died
  Analysis time _t: t
```



The graph shows that the predicted high-risk patients have a more steeply falling survival curve than the predicted low-risk patients. To confirm this conjecture, we do a log-rank test.

```
. sts test risk

        Failure _d: died
  Analysis time _t: t

Equality of survivor functions
Log-rank test

          |  Observed       Expected
risk      |    events         events
----------+--------------------------
 Low risk |        39          68.17
High risk |        51          21.83
----------+--------------------------
    Total |        90          90.00

                       chi2(1) =  61.50
                       Pr>chi2 = 0.0000
```

The log-rank test rejects the hypothesis that the predicted low-risk and high-risk patients have the same survival functions. Both the Kaplan–Meier plot and the log-rank test show that using the predicted hazard ratios' median can effectively distinguish a low-risk patient from a high-risk patient. We can now make prognostic predictions given new data.

The dataset (`newlungcancer.dta`) contains histopathology image features for some new stage I adenocarcinoma patients, but their survival time is not recorded because they are still alive. Based on the prediction model from `lasso cox`, we want to classify these new patients as low risk or high risk. To achieve this objective, we need to predict the new patients' hazard ratios and compare them with the median level of risk score obtained in the training data.

```
. use https://www.stata-press.com/data/r18/newlungcancer, clear
(Fictitious new data on stage I adenocarcinoma lung cancer)

. predict riskscore_new
(options hr penalized assumed; predicted hazard ratio with penalized
coefficients)

. generate risk = (riskscore_new >= $median)

. label define risk_lb 1 "High risk" 0 "Low risk"

. label values risk risk_lb

. tabulate risk

       risk |      Freq.     Percent        Cum.
------------+-----------------------------------
   Low risk |         27       54.00       54.00
  High risk |         23       46.00      100.00
------------+-----------------------------------
      Total |         50      100.00
```

The table of the predicted risk level shows that 27 patients are classified as low risk, while 23 patients are classified as high risk.

References

Sohn, I., J. Kim, S.-H. Jung, and C. Park. 2009. Gradient lasso for Cox proportional hazards model. *Bioinformatics* 25: 1775–1781. https://doi.org/10.1093/bioinformatics/btp322.

Yu, K., C. Zhang, G. J. Berry, R. B. Altman, C. Ré, D. L. Rubin, and M. Snyder. 2016. Predicting non-small cell lung cancer prognosis by fully automated microscopic pathology image features. *Nature Communications* 7(12474). https://doi.org/10.1038/ncomms12474.

Also see

[LASSO] **lasso** — Lasso for prediction and model selection

[LASSO] **lasso fitting** — The process (in a nutshell) of fitting lasso models